# Mandatory Exercise
IIAV3017 Advanved Control

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## Task1 (50%): LQ optimal control of continuous systems

Given a linear continuous time system described by the state space model

$$\dot{x} = Ax + Bu, \tag{1}$$
$$y = Dx, \tag{2}$$

where $x$ is the system state vector and $u$ is the control vector.
Consider a standard Linear Quadratic (LQ) optimal criterion or performance index

$$J = \frac{1}{2}\int_0^\infty [y^T Q y + u^T P u] dt, \tag{3}$$

where $Q$ and $P$ are weighting matrices.
Answer the following:

**a)** The optimal control $u^*$ is $u^* = Gx$. Find an expression for the state feedback matrix G ?

**b)** The feedback matrix $G$ is a function of the solution $R$ of an Algebraic Riccati Equation (ARE). Find an expression for the ARE ?

**c)** Related questions:

- Is the closed loop system $\dot{x} = Ax + Bu^* = (A + BG)x$ stable ?
- Is there any assumptions for the stability of the closed loop system ?
- Is the optimal control of state feedback type? or output feedback type?

## Task2 (50%): LQ optimal control of discrete time systems

Assume that the process may be described by a discrete time state space model

$$x_{k+1} = Ax_k + Bu_k, \tag{4}$$
$$y_k = Dx_k, \tag{5}$$

where $x_k \in \mathbb{R}^n$ is the state vector and $u_k \in \mathbb{R}^r$ is the control vector.
Consider a standard discrete time Linear Quadratic (LQ) optimal criterion

$$J_i = \frac{1}{2} \sum_{k=i}^{\infty} (y_k^T Q x_y + u_k^T P u_k), \tag{6}$$

where $Q$ and $P$ are weighting matrices.

**a)** The optimal control $u_k^*$ is $u_k^* = G x_k$. Find an expression for the optimal state feedback matrix G ?

**b)** The feedback matrix $G$ is a function of the solution $R$ of a Discrete Algebraic Riccati (DARE) equation. Find an expression for the DARE ?

**c)** Related questions:

- How many solutions does there exist of the DARE ?
- Should the solution of the DARE be positive, negative or indefinite ?
- Is the feedback matrix $G$ constant or time varying ?

# 1 Tip

Most of the solutions in Task1 and task2 above may be found in Lecture Notes Ch4.2, Ch4.4 and Ch5.2