IIA
EIK
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Advanced Control

Øving 9 (LQG, control and estimation)

Task 1

Given a SISO system described with the following state space model

$$\dot{x} = ax + bu + v \tag{1}$$
$$y = x + w \tag{2}$$

where x is the system state, y is the measurement, v is white process noise (zero mean with given variance), w is the measurement noise (white, zero mean with given variance). I.e.

$$E(v(t)) = 0 \quad \text{and} \quad E(v(t)v^T(t+\tau)) = q_0^2\delta(\tau) \tag{3}$$
$$E(w(t)) = 0 \quad \text{and} \quad E(w(t)w^T(t+\tau)) = r_0^2\delta(\tau) \tag{4}$$

where

$$\delta(\tau) = 1 \quad \text{for} \quad \tau = 0 \tag{5}$$
$$\delta(\tau) = 0 \quad \text{for} \quad \tau \neq 0 \tag{6}$$

Remarks
q_0^2 is the variance of the process noise v and q_0 is the standard deviation. I.e. the standard deviation is the square root of the variance
In case that v is a vector of process noise we have the covariance matrix ($E(v(t)v^T(t)) = V$). This matrix is positive definite $V > 0$ if one of the noise variables in v is not identical zero. If so, the covariance matrix V will be positive semi definite, i.e. $V \geq 0$.
The covariance matrix V may be Cholesky factorized if $V > 0$, so that $V = V_0V_0^T$ where V_0 is an upper triangular matrix This is (loosely) often referred to as square root factorization.
The standard deviation to the respective noise processes v_i in the vector v is then given as the square root of the diagonal elements in the covariance matrix V.
The diagonal element v_{ii} in the matrix V is the variance to the noise process v_i, where v_i is element number i in the vector v.

Assuming that v is given for a sequence of N discrete time instants then the covariance matrix may be computed/estimated as

$$V = \frac{1}{N} \sum_{t=0}^{N-1} v_t v_t^T \qquad \text{(biased estimate)} \tag{7}$$

$$V = \frac{1}{N-1} \sum_{t=0}^{N-1} v_t v_t^T \qquad \text{(unbiased estimate)} \tag{8}$$

1. Write up an optimal state estimator, i.e. a kalman filter, for the system. Use the duality principle and comment upon the similarity with LQ optimal control.

2. Commenting upon the solution as a function of the standard deviations q_0 and r_0.

3. Assume that we want an LQG controller. Discuss the principle of the LQG controller.

Task 2

Given a SISO one state system described by the model

$$\begin{aligned} \dot{x} &= ax + bu + cv & (9) \\ y &= x + w & (10) \end{aligned}$$

The model is the same as the model in Task 1 but the process noise have non-zero mean, i.e.

$$E(v) = \bar{v} \quad (11)$$

1. Assume that the noise v is slowly varying. The noise may then be modelled as a random walk, i.e.

$$\begin{aligned} v_{t+1} &= v_t + \Delta t dv && \text{discrete noise model} & (12) \\ \dot{v} &= dv && \text{continuous noise model} & (13) \end{aligned}$$

   where Δt is sampling time. Notice that the discrete model is found by Euler discretization of the continuous model. dv is a white noise process with zero mean and given variance q_0^2. Make an augmented model, i.e. combine the process model and the noise model for designing an optimal estimator.

2. Write up an optimal state estimator, a Kalman filter, for the system (the augmented system). Use an infinite time horizon, i.e. a steady state Kalman filter. Use with advantage *MATLAB* and the function **lqe**. Numerical values could be $a = -1$, $b = 0.5$, $c = 0.6$, $r_0 = 1$ and $0.01 \leq q_0 \leq 10$. Simulate the system and the state estimator with the function **dlsim** or **dsrsim**.)