EIK
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IIAV3017 Advanced Control

Øving 8

Task 1: Discret optimal control and tracking systems

Work through Task 1 Exam June 1997.

Task 2: Weighting control deviations

We are in this section to work through Example 5.2 in the lecture notes. The example ar about weighting control input deviations in connection with tracking systems.

a) Assume that we want $y = r$ in steady state. Find the corresponding values for x and u. Specify $r = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$ and use the model in Example 5.2. Note: Those steady state values for x and u are to be used as initial values for the simulations of the closed loop controlled system.

b) Write down the augmented state space model for $x_{k+1} = Ax_k + Bu_k$, $y_k = Dx_k$ og $u_k = u_{k-1} + \Delta u_k$, where Δu_k is treated as a new control action. Discuss the poles of the closed loop system.

c) Compute the solution, R, of the algebraic Riccati equation and the feedback matrix G. The weighting matrices are as in Example 5.2.

d) use the MATLAB script-files **main_dlq_rdu.m** and **main_dlq_rdu2.m**. Do some changes in the weighting matrices Q and/or $\mathcal{R}$ and study the responses in y_k and u_k. Modify e.g. such that $Q = qI$ and tune on q e.g. by using the **dread** function as follows,

```
q=0.03;                                          % Default value on, q.
q=dread('Weighting parameter q=',q);              % Gives new value on, q.
Q=q*eye(2);
```

Task 3: Optimal control of chemical reactor

A non-linear model for the chemical reactor is given by.

$$\begin{aligned} \dot{x_1} &= -k_1 x_1 - k_3 x_1^2 + (x_{10} - x_1)u, &(1)\\ \dot{x_2} &= k_1 x_1 - k_2 x_2 - x_2 u, &(2)\\ y &= x_2, &(3) \end{aligned}$$

where the reaction coefficients are given by $k_1 = 50$, $k_2 = 100$, $k_3 = 10$. The following steady state values for the states and the control are given, i.e. such that $\dot{x} = f(x, u) = 0$): $x_1^s = 2.5$, $x_2^s = 1$ and $u^s = 25$. The concentration of product A in the feed to the tank is assumed to be slowly varying or simply constant with value $x_{10} = 10$ for use in the simulation experiments.

a) Show that a linearized state space model is given by

$$\begin{aligned} \dot{x} &= A_c x + B_c u + C_c v, &(4)\\ y &= D_c x, &(5) \end{aligned}$$

where

$$A_c = \begin{bmatrix} -125 & 0 \\ 50 & -125 \end{bmatrix}, \; B_c = \begin{bmatrix} 7.5 \\ -1 \end{bmatrix}, \; C_c = \begin{bmatrix} 25 \\ 0 \end{bmatrix}, \; D_c = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad (6)$$

and

$$x := x - x^s, \; u := u - u^s, \; v := v - v^s, \; y := y - y^s, \quad (7)$$

and $y^s = Dx^s$.

b) The sampling time or step length is given by $h = 0.002$. Show that a discrete time state space model (where u is assumed constant over the sampling intervall) is given by

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Cv_k, &(8)\\ y &= Dx_k, &(9) \end{aligned}$$

where

$$A = \begin{bmatrix} 0.7788 & 0 \\ 0.0779 & 0.7788 \end{bmatrix}, \; B = \begin{bmatrix} 0.0133 \\ -0.0011 \end{bmatrix}, \; D = \begin{bmatrix} 0 & 1 \end{bmatrix}. \quad (10)$$

$$x_k := x_k - x^s, \; u_k := u_k - u^s, \; v_k := v_k - v^s, y_k := y_k - y^s. \quad (11)$$

c) We want to find and use a discrete controller which are minimizing the objective

$$J_i = \frac{1}{2} \sum_{k=i}^{\infty} (q(y_k - r)^2 + p\Delta u_k^2) \quad (12)$$

where $\Delta u_k = u_k - u_{k-1}$ is the control rate of change of the form

$$u_k = u_{k-1} + G_1 \Delta x_k + G_2(y_{k-1} - r), \tag{13}$$

where we have chosen the weights $q = 500$ and $p = 1$. Show that

$$G_1 = \begin{bmatrix} -23.4261 & -84.5791 \end{bmatrix}, \; G_1 = -20.0581. \tag{14}$$

d) Simulate the system given by (1)-(3) with the optimal control (13)-(14) after a step change in the reference signal r around the nominal reference value $r = 1$.

e) Find (or show) the similarity between an conventional PI controller on deviation form and the optimal controller in 13).

f) Simulate the system given by (1)-(3) with a conventional PI controller on deviation form after a step change in the reference signal r around the nominal value $r = 1$.

Use $K_p = 46.9$, $T_i = \frac{1}{83.3}$ which are one of the best PI settings found.

Tips: To this Task 3 there is created a MATLAB script, **dlq_ex3_du.m**.