Master study
IIA
EIK
University of South-Eastern Norway
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IIAV3017 Advanced Control with Implementation

Exercise 7

Task 1: Discretization

Work through Exercises 26.1 and 26.2 in the Lecture Notes: Obsolute

Task 2 2: Discrete LQ optimal control

Work through Example 5.1 in the Lecture notes., i.e. compute

- The solutions of the Riccati equation: R_0, R_1, R_2, R_3 and R_4.
- The feedback matrices: G_0, G_1, G_2, G_3 and G_4.
- the optimal control: u_0, u_1, u_2, u_3 and u_4.
- The states : x_1, x_2, x_3, x_4 and x_5.
- Simulate and plot the state and controls x_k, u_k.
- Plot the $R_{2,2}(k)$ element in R_k.

It is recommended that you first compute by hand and thereby write a MATLAB script for the computations.

Task 3: Discrete LQ-control

Given a system

$$x_{k+1} = ax_k + bu_k, \tag{1}$$
$$y_k = x_k. \tag{2}$$

and the following objective function

$$J_i = \frac{1}{2}sy_N^2 + \frac{1}{2}\sum_{k=i}^{N-1}(qy_k^2 + pu_k^2). \tag{3}$$

a) Show that the optimal control is given by

$$\begin{aligned} u_k &= g_k x_k, & (4) \\ g_k &= -\frac{abr_{k+1}}{p + b^2 r_{k+1}}, & (5) \end{aligned}$$

where r_{k+1} is defined by the discrete time Riccati equation

$$\begin{aligned} r_k &= q + a^2 r_{k+1} - \frac{a^2 b^2 r_{k+1}^2}{p + b^2 r_{k+1}}, & (6) \\ r_N &= s. & (7) \end{aligned}$$

Tips: take the general matrix Riccati equation as the starting point and substitue the scalar system parameters a and b and the criterion parameters q, p and s..

b) Use the numerical values $a = 0.9$, $b = 0.5$,$q = 2$, $p = 1$ and $s = 2$. Put $N = 10$, $i = 1$ and compute

- The optimal controls u_k, $k = 1, \ldots, 9$.
- the optimal outputs y_k, $k = 1, \ldots, 10$.
- The solution of the Riccati equation r_k, $k = 1, \ldots, 10$.
- The feedback parameters, g_k, $k = 1, \ldots, 9$.

This is most simply performed by using MATLAB. Plot the results with time.

Tips: se Example 9.1 for a solution sketch.