Master study
Systems and Control Engineering
Department of Technology
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SCEV3006 Advanced Control with Implementation

Exercise 6

Task 1

An angular positioning system is described by the model

$$\dot{x} = Ax + Bu, \quad y = Dx, \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; D = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{2}$$

An Linear Quadratic (LQ) objective criterion is given by

$$J = \int_{t_0}^{\infty} (y^T q y + u^T P u) dt = \int_{t_0}^{\infty} (x^T Q x + u^T P u) dt, \tag{3}$$

where

$$Q = q D^T D = \begin{bmatrix} q & 0 \\ 0 & 0 \end{bmatrix}, \; P = 1. \tag{4}$$

a) Investigate if the system is controllable. Which demands must be satisfied in order for an optimal control to exists which is guaranteed to stabilize the closed loop system?

b) Write down the Algebraic Riccati equation (ARE) and solve for the symmetric and positive definite solution R.

c) Show that the optimal control is given by

$$u^* = Gx, \tag{5}$$

where

$$G = \begin{bmatrix} -\sqrt{q} & -\sqrt{100 + 2\sqrt{q}} + 10 \end{bmatrix}. \tag{6}$$

d) Show that the poles of the closed loop system is given as the roots of the characteristic polynomial

$$\lambda^2 + (\sqrt{100 + 2\sqrt{q}})\lambda + \sqrt{q}. \tag{7}$$

d) Study the root locus of the poles as a function of the weighting parameter $q > 0$. Let q start at $q = 5000$ and let q decrease against zero, i.e. $q = 0$. Try to sketch the root locus in a MATLAB figure.

Task 2

We are in this exercise to study and control a heating tank with constant flow through the tank. The liquid which flows out of the tank should have temperature, x, as close to a constant temperature x_s as possible. The temperature on the liquid into the tank is equal to the surrounding temperature x_0. The heating power is simple equal to the control variable u. The flow trough the tank is constant and equal to q. The heating constant between the liquid and the surrounding is G. Furthermore, u_s, is steady state control when the temperature in the tank is equal to x_s and when the surrounding temperature is x_0. We have the following numerical values

$$\begin{array}{ll} q = 1 & [\frac{m^3}{h}] \\ V = 1 & [m^3] \\ C_p\rho = 800 & [\frac{Kcal}{°C m^3}] \\ G = 1600 & [\frac{Kcal}{°C}] \\ x_0 = 20 & [°C] \end{array}$$

We notice that we here have denoted the product $C_p\rho$ between the heat capacity C_p and the density ρ of the liquid.

a) Sketch a figure of the process where you marks into the defined variables and constants. Find a mathematical model of the process. Use an energy balance as the starting point.

b) Find the control u_s which gives a tank water temperature x_s when the surrounding temperature is x_0.

c) What is the basic steps in the maximum principles ?

d) We want to control the process such that the following criterion is minimized

$$J = \int_{t_0}^{\infty} (q(x - x_s)^2 + p(u - u_s)^2) \tag{8}$$

where the weights are specified to be $q = 1 \cdot 10^9$ and $p = 1$. Find the LQ optimal control u^*.

Task 3

Given a process

$$\dot{x} = u \tag{9}$$
$$y = x \tag{10}$$

The output y is to follow a prescribed reference r such that the criterion

$$J = s(r(t_1) - y(t_1))^2 + \int_{t_0}^{t_1} [q(r-y)^2 + u^2]dt \tag{11}$$

is minimized.

a) Find the solution to the above problem. You should define the necessary equations. Also try find the analytical solution.

b) We are choosing the parameters $s = 2$, $q = 1$, $t_0 = 0$, $x(0) = 0$, $t_1 = 10$. The reference r is a step at $r = 0$ for $t_0 \le t < 5$ and $r = r_0$ for $5 \le t \le t_1$ where $r_0 = 1$.

   Simulate the system in MATLAB. Use the explicit Euler method in order to integrate the necessary differential equations. The step length may e.g. be $\Delta t = 0.05$. Compare with the analytical solution.

   Tips: a m-file script **ov6oppg3.m** is available at the info server, the I: disc.

c) Discuss the stability properties of the LQ controller and the solution in step a) above. Is it possible to find a weight s such that the closed loop system is stable even for a finite horizon LQ problem with $0 < t_1 < \infty$?

d) We now chose $t_0 = t$ and $t_1 = t + T$ where T is the prediction horizon. is it possible to chose a weight s such that the closed loop system is stable independent of the choice of the prediction horizon T. Simulate the system with this weighting, if you have found one. use $T = 1$.