Master study
Systems and Control Engineering
Department of Technology
Telemark University College
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SCEV3006 Advanced Control with Implementation

Exercise 3

Task 1

We will in this task solve a Linear Quadratic (LQ) optimal control problem. Consider the following objective functional (criterion)

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T P u) dt \tag{1}$$

where

$$\dot{x} = Ax + Bu, \tag{2}$$

are to be minimized with respect to the control u. The initial state $x(t = 0) = x_0$ is assumed given.

a) Numerical values for this exercise are $A = -0.1$, $B = 1$, $Q = 1$, $P = 1$ and $x_0 = 5$. Find the optimal control u^* and the optimal feedback matrix G. This means that you should solve the optimal control problem described above. Simulate the closed loop system.

b) Find the optimal control u^* for the problem described above when the system matrices and the weighting matrices are as follows

$$A = \begin{bmatrix} -0.1 & 0 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad P = 1, \; x_0 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \tag{3}$$

You should now have found an optimal PI controller for the system in step 1a). What is the equivalent PI-controller paramaters K_p and T_i? Simulate the closed loop system. You may with advantage use a non-zero reference signal.

Tips: You may in this exercise use the Control System Toolbox function **lqr** in order to obtain the LQ optimal feedback matrices. However, the equations to be solved should be properly defined. Those equations may be solved by hand calculations

Task 2

In this task we will solve an LQ optimal control problem. The following performance index

$$J(G) = \int_0^7 (x^T Q x + u^T P u) dt \tag{4}$$

where

$$\begin{aligned} \dot{x} &= Ax + Bu, & (5) \\ u &= Gx, & (6) \end{aligned}$$

is to be minimalized with respect to the feedback gain matrix G.
The initial state vector $x(t = 0) = x_0$ is given.
In this task we will solve the optimal control problem by direct optimization theory without solving the Riccati equation.
a) Numerical values are specified as follows, $A = -0.1$, $B = 1$, $Q = 1$, $P = 1$ og $x_0 = 5$.
Find the optimal control feedback matrix G and simulate the closed loop system !
b) Find the optimal feedback gain matrix G when the model and weighting matrices are as follows

$$A = \begin{bmatrix} -0.1 & 0 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad P = 1, \; x_0 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \tag{7}$$

You should now have found an optimal PI controller for the system in step a) above. Compare with a conventional PI controller and indicate the corresponding $K : p$ and T_i parameters.
Simulate the closed loop controlled system, use e.g. an non-zero reference signal.
c) Compare the above results by using the MATLAB lqr functiom
Tips to steps a) and b): Use the MATLAB function **fminsearch**.