Sluttprøve i fag A3802 Avansert reguleringsteknikk med programmering torsdag 15. desember 2004 kl. 9.00 - 12.00

Sluttprøven består av: 3 oppgaver.
Oppgaven teller 70 % av sluttkarakteren.
Det er 3 sider i sluttprøven.
Tillatte hjelpemidler: vedlegg til oppgaven

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Task 1 (20%) (Continuous optimal control)

Given an LQ optimal control objective defined over a finite time horizon, i.e.,

$$J = \frac{1}{2}x(t_1)^T S x(t_1) + \frac{1}{2}\int_{t_0}^{t_1} [y^T Q y + u^T P u] dt, \tag{1}$$

where $S \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{m\times m}$ and $P \in \mathbb{R}^{r\times r}$ are symmetric weighting matrices.

a) Consider a given discrete state space model

$$\dot{x} = Ax + Bu, \tag{2}$$
$$y = Dx, \tag{3}$$

for a process. Find the optimal control which minimize the objective, J, given by (1) subject to the model (2) and (3).

The solution should consist of:

1. An expression for the optimal control vector, u^*.
2. A Riccati equation.
3. A final value condition for the Riccati equation.

b) Consider now an infinite horizon LQ control objective as follows

$$J_i = \frac{1}{2}\int_{0}^{\infty} [x^T Q x + u^T P u], \tag{4}$$

and the model as in (2) and (3) above.

What is now the solution to the LQ optimal control problem?

Task 2 (30%) (Discrete LQ optimal control with Integral Action)

We are in this task to study an LQ optimal controller for a system described by the state space model

$$x_{k+1} = Ax_k + Bu_k + v, \tag{5}$$
$$y_k = Dx_k + w, \tag{6}$$

where v and w are constant and unknown disturbances.
Subject to the above state space model we want to design an LQ optimal controller which minimizes the following LQ criterion

$$J_i = \frac{1}{2}\sum_{k=i}^{\infty}((r - y_k)^T Q (r - y_k) + \Delta u_k^T P \Delta u_k). \tag{7}$$

where $\Delta u_k = u_k - u_{k-1}$ and r is a constant reference vector. Q and P are symmetric and positive semidefinite matrices.

a) Show that it is possible to write the model in (5) and (6) on deviation form, i.e.,

$$\Delta x_{k+1} = A\Delta x_k + B\Delta u_k, \tag{8}$$
$$\Delta y_k = D\Delta x_k, \tag{9}$$

where

$$\Delta x_k = x_k - x_{k-1}, \;\; \Delta u_k = u_k - u_{k-1}, \;\; \Delta y_k = y_k - y_{k-1}. \tag{10}$$

What can be gained by doing this?

b) Show that the model in (8) and (9) can be written as follows

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}\Delta u_k, \tag{11}$$
$$\tilde{y}_k = \tilde{D}\tilde{x}_k, \tag{12}$$

where

$$\tilde{x}_k = \begin{bmatrix} \Delta x_k \\ r - y_{k-1} \end{bmatrix}, \;\; \tilde{y}_k = r - y_k. \tag{13}$$

Here you should define the matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{D}$.

c) The state space model in 2b) and the LQ criterion in (7) defines a standard discrete LQ optimal control problem of the form

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}\tilde{u}_k, \tag{14}$$
$$\tilde{y}_k = \tilde{D}\tilde{x}_k, \tag{15}$$

with LQ criterion

$$J_i = \frac{1}{2}\sum_{k=i}^{\infty}(\tilde{y}_k^T Q\tilde{y}_k + \tilde{u}_k^T P\tilde{u}_k), \tag{16}$$

where we for simplicity has defined

$$\tilde{u}_k = \Delta u_k. \tag{17}$$

Write down the solution to the LQ optimal control problem, of the form

$$\tilde{u}_k^* = \tilde{G}\tilde{x}_k. \tag{18}$$

the solution should consist of:

1. A discrete Riccati equation
2. an expression for the controller matrix $\tilde{G}$.
3. Write down an expression for the actual control of the form

$$u_k = f(\cdot) \tag{19}$$

which are to be used in order to control the process.

Task 3 (20%) (Diverse questions)

a) What is the meaning of the "duality principle" ? write down a table which illustrates the "duality principle" for continuous systems.

b) Given a linear system

$$\dot{x} = Ax + Bu \tag{20}$$

where $x(t_0)$ is a specified initial state vector.

Write down an expression for the solution, $x(t)$?

c) Given a PID controller

$$u(s) = K_p e(s) + \frac{K_p}{T_i s} e(s) + K_p T_d s e(s) \tag{21}$$

where $e(s)$ is the control deviation.

- Find a continuous state space model for the PID-controller. Note: it is not necessary to discuss problems involving $\dot{e}$ in the answer!
- Use the result from task 3b) above in order to find an expression for the solution, $u(t)$. Tips: Here we find an formulation of, $u(t)$, in terms of among others an integral.

d)

- What is an LQG controller? Sketch a block diagram for a system controlled by an LQG controller

Appendix

Continuous time optimal control

$$\begin{aligned}
\dot{x} &= f(x,u,t) && (22)\\
J &= S(x(t_1)) + \int_{t_0}^{t_1} L(x,u,t)dt && (23)
\end{aligned}$$

The continuous time Maximum Principle

$$\begin{aligned}
H &= L + p^T f && (24)\\
\dot{p} &= -\frac{\partial H}{\partial x} && (25)\\
p(t_1) &= \frac{\partial S}{\partial x}|_{t_1} && (26)
\end{aligned}$$

Discrete time optimal control

$$\begin{aligned}
x_{k+1} - x_k &= f(x_k,u_k,k) && (27)\\
J_i &= S(x_N) + \sum_{k=i}^{N-1} L(x_k,u_k,k) && (28)
\end{aligned}$$

The discrete time maximum Principle

$$\begin{aligned}
H_k &= L(x_k,u_k,k) + p_{k+1}^T(x_{k+1} - x_k) && (29)\\
p_{k+1} - p_k &= -\frac{\partial H_k}{\partial x_k} && (30)\\
p_N &= \frac{\partial S}{\partial x_N} && (31)
\end{aligned}$$

Derivation rules

$$\frac{\partial}{\partial x}(Qx) = Q^T, \quad \frac{\partial}{\partial x}(x^T Q x) = Qx + Q^T x \tag{32}$$

$$\frac{\partial}{\partial x}((r - Dx)^T Q (r - Dx)) = -2D^T Q(r - Dx) \tag{33}$$