Non-linear System Identification

Dr. David Di Ruscio
Telemark University College
Systems and Control Engineering
N-3914 Porsgrunn, Norway
Fax: +47 35 57 54 01
Tel: +47 35 57 51 68
Email: david.di.ruscio@hit.no
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Abstract

Some techniques for system identification of non-linear systems are addressed.

1 Non-linear dynamic systems

1.1 Convexification

By convexification we mean to reformulate a non-convex optimization problem as a convex optimization problem which may be solved as an Ordinary Least squares (OLS) regression problem. Hence, many non-linear regression problems may be formulated as the linear regression model

$$y_k = \varphi_k^T \theta + e_k, \tag{1}$$

where y_k is a measured quantity or measured variables, also denoted the regressed variables. φ_k is a vector/matrix of known elements and often defined as regression variables or regressors. θ is the vector of unknown parameters.

We will in the following present some examples of non-linear identification problems which may be convexified, i.e., formulated as a convex linear regression problem.

Example 1.1
Given a system described by

$$\dot{y} = \theta_1 \frac{y}{\theta_2 + y} - y + u, \tag{2}$$

with discrete time observations

$$y_k^m = y_k + w_k. \tag{3}$$

Here, y_k^m*, is the measured observation of* $y = y(t)$ *at discrete time,* k*.* w_k *represents measurements noise.*

The problem of minimizing the prediction error criterion

$$V_N(\theta) = \sum_{k=1}^{N} (y_k^m - \bar{y}_k^m)^2 \tag{4}$$

is a non-linear non-convex optimization problem in the parameter vector $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$*. Here the prediction* $\bar{y}_k(\theta)$ *of* $y = y(t)$ *is the solution of Eq. (2).*

Multiplying Eq. ((2) with $\theta_2 + y$ *and rearranging gives the least squares problem*

$$y\dot{y} + y^2 - uy = \begin{bmatrix} y & u - y - \dot{y} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}. \tag{5}$$

Hence, the parameters θ_1 *and* θ_2 *may simply be found from a least squares regression problem. Here, we have convexified the non-convex original non-linear problem.*

Example 1.2
Given a system described by

$$y_k = \theta u_k + \theta^2 u_{k-1} + w_k. \tag{6}$$

This model may be rearranged and described linear in the parameter θ *as*

$$y_k u_{k-2} - y_{k-1} u_{k-1} = \theta(u_k u_{k-1} - u_{k-1}^2) + e_k, \tag{7}$$

where the equation error, e_k*, is*

$$e_k = w_k u_{k-2} - w_{k-1} u_{k-1}. \tag{8}$$

Eq. (7) may be deduced as follows. Express Eq. (6) at time $k-1$*, i.e.,*

$$y_{k-1} = \theta u_{k-1} + \theta^2 u_{k-2} + w_{k-1}. \tag{9}$$

Multiply Eq. (6) with u_{k-2} *and Eq. (9) by* u_{k-1}*, and we obtain the equations*

$$u_{k-2} y_k = \theta u_k u_{k-2} + \theta^2 u_{k-1} u_{k-2} + w_k u_{k-2}. \tag{10}$$

$$u_{k-1} y_{k-1} = \theta u_{k-1}^2 + \theta^2 u_{k-2} u_{k-1} + w_{k-1} u_{k-1}. \tag{11}$$

Subtracting Eq. (11) from Eq. (10), i.e., express $u_{k-2} y_k - u_{k-1} y_{k-1}$*, gives Eq. (7)*

2 Steady state systems

Example 2.1
The Antoine equation

$$p = 10^{A - \frac{B}{C+T}} \tag{12}$$

or equivalently

$$log_{10}(p) = A - \frac{B}{C+T} \tag{13}$$

is a vapor pressure equation and describes the relation between vapour pressure, p, and temperature, T, for pure components. A, B and C are component-specific constants.

Multiplying Eq. (13) with $C+T$ and rearranging gives the linear regression model

$$T log_{10}(p) = \begin{bmatrix} log_{10}(p) & T & 1 \end{bmatrix} \begin{bmatrix} -C \\ A \\ AC - B \end{bmatrix} \tag{14}$$