

# DEVELOPMENT OF SYSTEM IDENTIFICATION

Lennart Ljung

*Department of Electrical Engineering, Linköping University  
Linköping, Sweden. Email: Ljung@isy.liu.se*

## 1. INTRODUCTION

To write a “history of system identification” is a formidable task and requires skills in science of history which I do not have. It is therefore unavoidable that an essay like this will be strongly subjective, and reflect my background, my personal interests and my personal recollections. The limitations in this presentation are thus set by my own background. I joined the Division of Automatic Control at Lund University in Sweden as a Ph.D. student in the summer of 1970. This was immediately after the second IFAC Symposium on Identification in Prague. At that conference, Åström and Eykhoff had just presented their survey paper (Åström and Eykhoff 1971), which later appeared in *Automatica*, and became a very much cited reference for system identification.

I started to do research on persistently exciting signals in 1971, at the same time as I studied Tsytkin’s then fairly recent work on stochastic approximation for recursive estimation and “self learning”. Adaptive – or self tuning – control was then the hot subject at Lund and elsewhere in the control community, and much of my own interest was focused around the problem of establishing convergence of adaptive control algorithms. That led me both to analysis of recursive algorithms and to consistency questions for various identification schemes, which in turn resulted in my Ph.D. thesis 1974. That sets the stage for my interest in system identification and this also gives the limitations of the present paper.

There are few accounts of the history of system identification in the literature. In addition to literature overviews in textbooks and survey papers, the only really “historic” article I know of is the one by Pieter Eykhoff in the *Systems and Control Encyclopedia*, (Eykhoff 1987).

## 2. SYSTEM IDENTIFICATION AS A RESEARCH FIELD IN AUTOMATIC CONTROL

The term *system identification* was coined by Lotfi Zadeh in 1962, (Zadeh 1962). He defined system identification as:

Identification is the determination, on the basis on input and output, of a system within

in a specified class of systems, to which the system under test is equivalent.

This definition is of course highly systems oriented, and does not really reflect the strong statistical flavor of system identification techniques. Nevertheless, the term caught on and soon became the standard terminology in the control community. On the other hand it has not spread outside our own community. In statistics, econometrics, geophysics, signal processing, etc., where also models of dynamic systems are build based on observed input output data, other terms are used.

Of course, system identification techniques have been applied as long as we have had feedback control. Transient response analysis was actively used to tune PID controllers; frequency analysis was used in conjunction with classical Bode-Nichols synthesis etc. It was however only with the advent of the so called modern control theory around 1960 with explicit use of parametric models, that more substantial activities in estimating systems began. The construction of models did not pose itself as a research challenge of the character “unsolved problem”, since several existing techniques in the statistical field for time series could rather easily be adopted. The early sixties were characterized by the development of a variety of different approaches to parameter estimation in dynamic systems and estimating linear systems in general. It could be said that system identification was established as a certified research field within the automatic control area in the middle of the sixties: At the third IFAC Congress in London, 1966 a survey paper, (Eykhoff *et al.* 1966) on system identification was presented. A year later, 1967, the first IFAC Symposium on system identification was organized in Prague. This is now the longest running IFAC symposium series.

Since then system identification has been an established field of automatic control with regular sessions at all general control meetings like the CDC and IFAC Congress, with special issues occuring now and then in the major control journals etc. The number of papers on system identification related problems in international control oriented conferences and journals must be of the order  $10^5$ .

### 3. THE STATISTICAL ROOTS OF SYSTEM IDENTIFICATION

A fair amount of system identification techniques and theory rests upon a statistical foundation. In fact, the inclusion of an input signal into time series analysis, and the formulation of the parameter estimation in dynamic systems as non linear regressions (or even linear regressions) is not particularly deep nor difficult.

This means of course that people like K. F. Gauss (Gauss 1809) and R. A. Fisher (Fisher 1912) and many other famous mathematicians and statisticians have also laid the foundations for system identification.

A full fledged statistical perspective was brought into the system identification field, perhaps first in the paper Åström and Bohlin who developed a maximum likelihood method for ARMAX models in (Åström and Bohlin 1965). A witness of this is for example that ten out of twenty seven references in this paper are solid statistical ones.

Otherwise, I think it must be said that the interaction between the statistics area and system identification field has been remarkably insignificant, in view of the very close relationship between time series analysis and non linear regression on one hand and system identification on the other. A typical paper today on system identification would have much fewer references to the statistical area than the just mentioned one.

Relatively few leading statisticians have taken part in the development of system identification: Manny Parzen, Ted Hannan and Hiro Akaike are three significant exceptions. The famous book by Box and Jenkins (Box and Jenkins 1970) has had a substantial influence in many areas of engineering, but perhaps not as much in the control area, despite that it actually partly deals with control problems. A possible explanation is the division of the publication and conference area: as far as I know, George Box has not participated in a control meeting or published any paper in a control oriented journal. On the other hand Akaike participated in the IFAC Congress in Helsinki and also written frequently in control oriented journals. Ted Hannan had many personal contacts with people in the control area, and also closely followed the system identification literature. Manny Parzen took part. e.g., in the IFAC Symposium in York.

### 4. THE UPS AND DOWNS OF SYSTEM IDENTIFICATION

System identification represents the interface between the world of mathematical models and the real world.

As such it is and will remain a fundamental problem area. Any application of control theory to the real world must in one way or another deal with the system identification problem. On the other hand, system identification has never really posed any well defined, open problems, that have attracted many researchers simultaneous interest. (A noteworthy exception to this is the so called errors-in-variable problem, as vigorously pointed out and treated by Kalman, e.g., (Kalman 1983). The errors-in-variable problem concerns the problem where both the input and output signals are measured with errors of unknown character.) It is also an area with a fairly low “entry threshold”: unlike, for example, non linear control theory, there is not a heavy mathematical machinery that has to be mastered in order to invent and check out new estimation methods.

Perhaps this is the reason why the “status” of system identification research has gone up and down over the years. The first decade of system identification as an established field, mid sixties to mid seventies, saw a very intense development period and a lot of corresponding general interest in the area. (Nevertheless even at the 1970 Prague symposium on identification, Åström and Eykhoff felt it necessary to defend research in the area. They end their survey paper by the sentence: “Also after the IFAC Prague, 1970 Symposium much work remains to be done.”)

During the 1980:s, though, there is a clear decline in the interest of system identification. It could be evidenced by looking at the number of sessions at international symposia and number of papers published in the journals. I could also exemplify this by the following: IFAC’s Technical Board – which I was a member of 1984 - 1993 – wanted to show renewal initiative by not only creating new symposium series, but also discontinuing some. At that point the system identification symposia were pointed out as a candidate to be abolished since “the interest anyway had moved to adaptive control”.

In view of this, the late eighties and the nineties have shown a considerable rebirth of development of system identification techniques. More about that later, but I could just mentioned that the first plenary on System Identification at an IFAC Congress was delivered in 1993 at the Sydney World Congress.

### 5. SYSTEM IDENTIFICATION AND IFAC

The development of system identification has many IFAC links. The most important of these is no doubt the series of symposia on system identification that has been arranged since 1967. The list of organizers of this event

also contain many of the well known contributors to the area.

What was the “hot topics” during these different conferences? Well, a sample of the proceedings, in conjunction with personal and subjective recollections could give the following list (which should be taken with a grain of salt).

- (1) Prague, 1967 (V. Strejc and V. Peterka): *New methods*.
- (2) Prague, 1970 (V. Strejc and V. Peterka): *New methods and need for unification*.
- (3) The Hague, 1973 (P. Eykhoff): *Comparisons of methods*.
- (4) Tbilisi, 1976 (N.S. Rajbmann): *Identification in closed loop*.
- (5) Darmstadt, 1979 (R. Isermann): *The intended use of the model*.
- (6) Washington D.C., 1982 (G. Bekey and G. Saridis): *Non-technical Applications*
- (7) York, 1985 (H.A. Barker and P.C. Young): *Adaptation, Identifiability*
- (8) Beijing, 1988 (H.F. Chen and B. Liu): *Adaptation, Signal Processing Applications*
- (9) Budapest, 1991 (Cs. Bányász and L. Keviczky): *Identification for control, New noise models*.
- (10) Copenhagen, 1994 (M. Blanke and T. Söderström): *Subspace techniques, New non-linear model types*.

There is a clear dominance of Europe in this list, and it is also a fact that European researchers have been, relatively speaking, very active in the area of system identification.

## 6. PERIODS IN THE DEVELOPMENT OF SYSTEM IDENTIFICATION

With a considerable amount of simplification one can distinguish the following periods in the development of system identification:

- (1) ... -1960: Development of the statistical roots.
- (2) 1960-1970: Proliferation of identification methods.
- (3) 1970-1985: Consolidation of system identification theory and practice.
- (4) 1985- ...: Emerging new ideas without statistical roots.

With a very crude approximation, the work in the different periods can be summarized as follows:

### 6.1 -1960: The statistical prehistory

This period essentially starts with Gauss, (Gauss 1809), and ends around 1960, when explicit parametric models become a major concern in the control community. During this period all the essential statistical concepts used in System Identification were developed. Of particular use is of course linear regressions and the Least Squares method, and its application to AR-models, (Mann and Wald 1943). The concepts, tools and analysis for non-linear regressions and the Maximum likelihood method (Fisher 1912), (Wald 1949), (Cramér 1946), naturally also belong to the foundations of System Identification.

Stochastic approximation, (Robbins and Monro 1951), was developed in the early 50-ies, and would later turn out to be a great source of inspiration for all on-line (recursive) identification techniques.

Several aspects of System Identification are really just variants of *Time Series Analysis*, and the vigorous development both of spectral methods and parametric methods for time series starting from Yule (Yule 1927), would have a profound impact on our field. At the end of this period the very influential books (Grenander and Rosenblatt 1957) and (Blackman and Tukey 1959) had been published and somewhat later came (Whittle 1963), (Hannan 1970), (Box and Jenkins 1970) and (Jenkins and Watts 1968), which all meant a lot to researchers in the identification area.

The parallel development in Econometrics for estimating models of economic dependencies for some reason has had less impact on the Identification field. An exception is the Instrumental Variable method for linear regressions, (Reiersøl 1941), which has been very popular, also in control applications.

### 6.2 1960–1970: Proliferation of identification methods

The status of the identification field 1970 was described by Åström and Eykhoff in (Åström and Eykhoff 1971) in the following way:

“The field of identification is at the moment rather bewildering, even for the so-called experts. Many different methods are being analysed and treated. “New methods” are suggested *en masse*, and, on the surface, the field looks more like a bag of tricks than a unified subject.”

To be true, this survey paper cites 230 references, virtually all from 1960-1970. In addition, there had been the survey papers, (Balakrishnan and Peterka 1969),

(Eykhoff 1968), (Cuenod and Sage 1968) and (Eykhoff *et al.* 1966) with another few hundred publications from this decade.

What was the reason for this explosion of methods? We can point to a few facts:

- It was immediate that the basic linear difference equation for input–output relationships could be written as a linear regression and that hence the Least Squares method could be applied. It was also soon clear that this led to biased estimates, except under very beneficial noise situations. That opened up an area for systematic approaches, as well as a number of tricks to deal with this bias. This led to methods like "The tally principle", "The extended matrix method", "Generalized Least Squares", "The instrumental Variable method", "Repeated Least Squares", "Extended Least Squares", "Panuska's method", "The maximum likelihood method" (which for a long time was essentially reserved for ARMAX models, in the System Identification literature), etc.
- Spectral and correlation techniques for time series were quite well developed, and it was natural to use and adapt these for the estimation of control systems.
- In addition to these statistics-oriented approaches, it was also natural to take a systems' oriented view, and start with the basic convolution relationship between input and output. Several techniques for deconvolution, realization, and function expansion of the impulse response were developed.

Out of these many attempts, it was no doubt the porting of the maximum likelihood method to dynamical systems that would have the strongest impact on the field in the long run. The application to ARMAX models in (Åström and Bohlin 1965) contains a complete statistical setup, with a systematic approach to estimation, including an asymptotic analysis of the estimate's properties.

The time was not really ripe for text books yet. One of the first books that dealt with System Identification was (Lee 1964). It was followed by (Eykhoff 1974), (Mendel 1973), which very well sum up the developments of this period.

### 6.3 1970-1985: Consolidation of system identification theory and practice

The survey paper (Åström and Eykhoff 1971) ends with a wish that more efforts be spent on unification and comparisons. To a large extent, that was also what would follow. Two main lines can be distinguished: Attempts

to see the connections between the different approaches, and more serious software work.

In the late sixties and early seventies, there was much talk about the necessity to compare the many different methods, but not much was done. (Usually it was the author's own method that turned out to be the best one.) An important reason for this was the difficulty to exchange programs and data. (I spent four months in 1971 just to convert a bunch of punched cards – Stanton's turbine alternator data – to a readable tape.) The computer and software development allowed and led to user friendly (i.e., could be used by others than the author) software packages for identification. One of the first and best known such interactive packages from this period was IDPAC, (Åström 1983), developed at Lund, but many similar and related ones were developed at universities with identification research. In addition to allowing serious industrial applications, this development had the important side effect that the researchers started to understand each others' methods.

To make a long story short, the essence of the difference in the methods was that they corresponded different noise assumptions and model structures, rather than being "different methods". This allowed substantial simplification and contraction in describing the field. It also shed light on the "comparisons": While it sounds reasonable to ask "Which is the better method?", it does not make sense to ask "Which is the better model, in general?" One simply has to have a variety of models on one's toolbox, and test them out on the actual data set.

This contraction was perhaps most pronounced for on-line, recursive estimation algorithms, – "A Fiddler's Paradise," according to (Åström and Eykhoff 1971) –, as summarized in (Ljung and Söderström 1983). The unifying view of System Identification as "*non-linear regression applied to dynamical systems*" is also the backbone of the textbooks (Ljung 1987) and (Söderström and Stoica 1989).

Experiment design and the influence of the experimental conditions on the identification result (including identification in closed loop) was another important topic in this period. Both could well be treated within the statistical framework: simply put, it is a question of computing and analysing the Fisher Information matrix. An influential textbook from this period, with a particular emphasis on experiment design, is (Goodwin and Payne 1977).

#### 6.4 1985 – : *Emerging new ideas without statistical roots.*

At the mid 1980:ies the statistical view of System Identification had matured and settled. The traditional and classical framework of parameter estimation had been succesfully and coherently ported to the world of dynamic systems. (I would like to stress *traditional and classical*; most of the more recent and advanced concepts in statistical inference have still not found their way to System Identification.) The corresponding methods had been found to be powerful and practical tools, and commercial software packages started to appear. Was then the area dead as an exciting research field? Even if some people thought so at the time, it turned out not to be the case.

What was awaiting was a number of different topics that had little or nothing to do with statistics. Some of the more important ones can be listed as follows: (I do not give any references in this section – this essay is not intended to be a survey of the current status of the field.)

*Subspace Methods for State-Space Models.* The systems oriented approach to identification – realization and deconvolution – had met with limited early success. However, by the early 90's, a realization based approach to esitimating state-space models (i.e., first find the states from data, and only then estimate the system) – often now called *subspace method* – had transpired to become a most effective and useful method, in particular for multivariable systems. This is in my mind the most interesting development in the past decade.

*Identification for Control.* Parallel with the development of robust control theory came some criticism of System Identification for not providing relevant model input for control design. (Part of that criticism was in my mind unfair and based on an incorrect understanding of the *model validation* process and the notion of confidence intervals.) That criticism has lead to a variety of activities:

- Evaluate the model properties in the frequency domain
- Device iterative schemes for experiment design, based on the outcome of previous experiments
- Seek alternative ways of describing model errors and disturbances
- Look into interpolation properties of the frequency function ( $H^\infty$ -identification.)

*Rejecting averaging properties of the noise* Averaging (ergodic) properties of the noise source (in particular be-

ing uncorrelated with input and/or reference signals) is at the heart of the statistical approach. It is not unnatural to question such ideal averaging features. This has lead to several different developments: The "unknown-but-bounded" or "set-membership" approach, and other algorithms that are robust to malign noise sequences ("worst case behaviour").

*Non-linear black box models* From a non-linear regression perspective there is basically no difference in estimating a linear or a non-linear system: we just need some parametrization (function expansion) of the predictor. The early approaches using Volterra (Taylor) expansion had met mixed success. The "new" world of expansions in terms of Neural Networks, Wavelet transforms, Fuzzy models, etc, are important also for dynamic systems.

*Frequency domain data* In many application areas, in particular in mechanical systems, it is natural to collect and store input-output data in the frequency domain. There is a very interesting development of techniques to work directly with such data.

## 7. SOME FURTHER REMARKS

System identification may seem to have had a slow and steady development. I have not been able to point to any big breakthroughs that have "changed the world". In fact our way of actually solving our identification problems today are not all that different from how Gauss solved his – it is just that we have a larger collection of model structures and better support, computationally and methodically.

The computational development has of course made it possible to carry out identification tasks that would have been intractable otherwise, but it has had a rather minor influence on the actual development of identification methods. (An exception are the sub-space methods that are closely linked to computational linear algebra.) Another side of the computing progress is that we can now store and efficiently work with very large data sets. I don't think that we have yet seen the impact of that technology development on the System Identification area.

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