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Multivariable identification of an activated sludge process with subspace-based algorithms

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Abstract

This paper is aimed at identifying a linear time-invariant dynamical model (LTI model with lumped parameters) of an activated sludge process. Such a system is characterized by stiff dynamics, nonlinearities, time-variant parameters, recycles, multivariability with many cross-couplings and wide variations in the inflow and the composition of the incoming wastewater. In this simulation study, a discrete-time identification approach based on subspace methods is applied in order to estimate a nominal MIMO state-space model around a given operating point, by probing the system in open-loop with multi-level random signals. Six subspace algorithms are used and their performances are compared based on adequate quality criteria, taking into account identification/validation data. As a result, the selected model is a very low-order one and it describes the complex dynamics of the process well. Important issues concerning the generation of the data set and the estimation of the model order are discussed.

Keywords: Subspace methods; Multivariable identification; State-space models; Activated sludge process; Environment engineering

1. Introduction

Advanced engineering applications require suitable mathematical models. System identification deals with the problem of obtaining models of dynamic systems from measured input–output data. This issue is of interest in a variety of applications, ranging from chemical process simulation and control to identification of vibrational modes in flexible structures. The most traditional system identification techniques are the prediction error method (PEM) and the instrumental variable method (IVM). These methods are primarily used with the so-called black-box model structures (Viberg, 1995).

The field of linear system identification is by now quite advanced. The traditional identification techniques offer good solution to many real-world problems. However, several important problems remain to be solved. The PEM has excellent statistical properties provided the "true" PEM estimate can be found. Nevertheless, computing the PEM model can sometimes be overwhelmingly difficult. In general, a multi-dimensional nonlinear optimization problem must be solved. On the other hand, the IVM attempts to deliver parameter estimates by only solving linear equation systems. However, the use of these models is quite cumbersome in the general multivariable case, and the numerical reliability may be unacceptably high for complex cases involving large system orders and many outputs (Viberg, 2002). The preferred model structure for complex problems is therefore a state-space model.

Subspace-based system identification method is a branch that has been recently developed in system identification (around 10 years old by now), which has attracted much attention, owing to its computational simplicity and effectiveness in identifying dynamic state-space linear multivariable systems. These algorithms are numerically robust and do not involve nonlinear optimization techniques, i.e., they are fast (noniterative) and accurate (since no problems with local minima occur). The computational complexity is modest compared to PEM, particularly when the number of

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inputs and outputs is large. Because applications of large dimensions are commonly found in the process industry, subspace identification methods are very promising in this field. As a result, a large number of successful applications of subspace identification methods for simulated and real processes have been reported in the literature.

In this paper, a low-order LTI discrete-time state-space multivariable model that describes the nitrate concentrations in the anoxic and aerobic zones of an activated sludge process (ASP) is estimated around an operating point. Several subspace identification methods are applied and their performances are compared in order to select the best-obtained model. It can be used to control the process, e.g., as in Lindberg (1997), where a multivariable control algorithm based on a subspace model is used to regulate an ASP. Previous performance comparisons of several subspace methods, applied to other processes, can be found in Abdelghani, Verhaegen, Van Overschee, and De Moor (1998), Katayama, Omori, and Picci (1998), Favoreel, Van Huffel, De Moor, and Sima (1999) and Juricek, Seborg, and Larimore (2001).

The structure of the paper is as follows: in the next section the main issues related to subspace identification methods and the subspace algorithms to be used in this study are shown. Section 3 briefly presents the ASP. Section 4 discusses the important steps in developing a suitable subspace model for the process and compares the performance of six subspace algorithms. Finally, Section 5 provides conclusions.

In this work, the ASWWTP-USP (Activated Sludge Wastewater Treatment Plant—University of São Paulo) benchmark (Sotomayor, Park, & Garcia, 2001a) is used as a data generator. This benchmark simulates the biological, physical and biochemical interactions that occur in a complex activated sludge plant.

2. Subspace identification methods

The discrete-time subspace identification methods refer to a class of algorithms whose main characteristic is the approximation of subspaces generated by the rows or columns of block-Hankel matrices of the input–output data, to calculate a reliable discrete-time state-space model of the following form:

$$x_{k+1} = Ax_k + Bu_k + \zeta_k,$$
$$y_k = Cx_k + Du_k + \eta_k, \tag{1}$$

with

$$\mathrm{E}\left[\begin{pmatrix}\zeta_p\\ \eta_p\end{pmatrix}\begin{pmatrix}\zeta_q^{\mathrm{T}} & \eta_q^{\mathrm{T}}\end{pmatrix}\right] = \begin{pmatrix}Q & S\\ S^{\mathrm{T}} & R\end{pmatrix}\delta_{pq} \geqslant 0, \tag{2}$$

where $x_k \in \Re^{nx1}$ represents the states vector, $u_k \in \Re^{mx1}$ is the manipulated input vector and $y_k \in \Re^{lx1}$ is the process output vector. $A \in \Re^{nxn}$ is the system (state transition) matrix, $B \in \Re^{nxm}$ is the input matrix, $C \in \Re^{lxn}$ is the output matrix and $D \in \Re^{lxm}$ is the direct input to output matrix. $\zeta_k \in \Re^{nx1}$ is called the process noise and $\eta_k \in \Re^{lx1}$ is called the measurement noise. They are assumed to be unmeasurable gaussian-distributed zero-mean, stationary white noise vector sequences. The matrices $Q \in \Re^{nxn}$, $S \in \Re^{nxl}$ and $R \in \Re^{lxl}$ are the covariance matrices of the noise sequences ζ_k and η_k. E denotes the expected value operator and δ_{pq} the Kronecker delta. The time index k denotes a discrete (sampled) system.

In subspace identification, it is typically assumed that the number of available samples goes to infinity, and that the data is ergodic. The following assumptions have been considered related to (1):

- the system is asymptotically stable;
- the pair (A, B) is controllable; and
- the pair (A, C) is observable.

It is common practice to distinguish among the three possible situations regarding the inputs acting on the system:

- the purely deterministic case ($\zeta_k = \eta_k = 0$);
- the purely stochastic case ($u_k = 0$); and
- the combined deterministic/stochastic case.

Subspace-based methods for state-space modeling have their origin in state-space realization, as developed by Ho and Kalman (1966). These techniques determine a state-space model from a given impulse response, which received a tremendous attention in the signal processing area in the late 1970s. In the system identification area, one usually has available input–output data rather than measured impulse response. In this context, subspace methods were developed in the late 1980s.

The term "subspace identification method" was first introduced by Verhaegen and Deprettere (1991). There are now many different versions of subspace methods. These include an early version of subspace algorithm presented in the paper by Moonen, De Moor, Vandenberghe, and VandeWalle (1989), Canonical Variate Analysis (CVA) by Larimore (1983, 1990), Multivariable Output-Error State-sPace model identification (MOESP) by Verhaegen and Dewilde (1992), Instrumental Variable Subspace-based State-Space System IDentification (IV-4SID) by Ottersten and Viberg (1994), Numerical algorithm for Subspace State Space System IDentification (N4SID) by Van Overschee and De Moor (1994, 1996), Canonical Correlation Analysis (CCA) by Peternell, Scherrer, and Deistler (1996) and Deterministic and Stochastic subspace system identification and Realization (DSR) by Di Ruscio (1997).

All subspace identification methods consist of three main steps: estimating the predictable subspace for multiple future steps, then extracting state variables from this subspace and finally fitting the estimated states to a state-space model. Nevertheless, each subspace identification method looks quite different from others in concept, computational tools and interpretation. The major differences among these subspace identification methods lie in the regression or projection methods used in the first step to remove the effects of the future inputs on the future outputs and, thereby, estimate the predictable subspace, and in the latent variable methods used in the second step to extract estimates of the states. A general overview of the state-of-the-art in subspace identification methods is presented in De Moor, Van Overschee, and Favoreel (1999) and Favoreel, De Moor, and Van Overschee (2000).

The major advantages of these algorithms are that they only need input–output data and very little prior knowledge about the system. In addition, these algorithms are based on system theory, geometry and numerically stable noniterative linear algebra operations, such as QR (or LQ)–factorization, SVD (singular value decomposition) and its generalizations, for which good numerical tools are well-known (Golub & VanLoan, 1996). A drawback against subspace identification approach is that the physical insight of the process, in the obtained model, is lost, which is a characteristic of black-box models. For example, the states are "artificial" and it is not possible to understand how a process variable, which is not directly included in the model, affects the process. Furthermore, a large amount of data is required to obtain accurate models. Actually, generating and collecting data of some processes can be too expensive. Important issues involved in developing a model through subspace identification methods can be found in Amirthalingam and Lee (1999).

Subspace identification methods have recently reached a certain level of maturity. The subspace identification algorithms considered in this paper are:

CCA: unconstrained CCA algorithm (uCCA), essentially the same CVA algorithm proposed by Larimore (1983); and constrained CCA algorithm (cCCA), proposed by Peternell et al. (1996).

MOESP: refined version of the past outputs (PO) scheme of the MOESP algorithm in the SMI Toolbox by Haverkamp and Verhaegen (1997).

N4SID: N4SID function (n4sid.m) in the MATLAB System Identification Toolbox v.4.0.4 (Ljung, 1997), that implements the N4SID algorithm from Van Overschee and De Moor (1994) and the "robust" N4SID algorithm from Van Overschee and De Moor (1996).

DSR: DSR algorithm in the D-SR Toolbox by Di Ruscio (1997).

As previously mentioned, the purpose of the present paper is to compare the performance of these methods and not to analyze their implementational differences. As for the detailed algorithms, the differences between these subspace identification methods seem to be so large that it is hard to find the similarities between them. Nevertheless, Van Overschee and De Moor (1995) showed that the subspace methods CVA, MOESP and N4SID are actually related to each other and that they differ only in the choice of weighting functions in a minimization problem. Di Ruscio (2000) reports the main differences and similarities among the algorithms CVA, MOESP, N4SID and DSR.

3. Description of the process

The ASWWTP-USP benchmark is a dynamic model, developed to simulate the processes that occur in a biological wastewater treatment plant (WWTP). The benchmark represents a continuous-flow pre-denitrifying ASP, a frequently applied system for removal of organic matter and nitrogen from municipal effluents, predominantly domestic, operating at a constant temperature of 15°C and neutral pH. The layout of the process is shown in Fig. 1.

The process configuration is formed by a bioreactor composed of an anoxic zone (zone 1 with 13 m^3), two aerobic zones (zones 2 and 3 with 18 and 20 m^3, respectively) and a secondary settler (20 m^3). In nominal steady-state conditions, the influent rate of raw wastewater is $Q_{in} = 4.17\,m^3/h$, with an average proportion of 224 mg COD/l of biodegradable organic matter and 44.88 mg N/l of total Kjeldahl nitrogen (TKN) and a hydraulic retention time of 17.0 h (based on total volume, i.e. bioreactor + secondary settler). The internal recycle flow rate is $Q_{\text{int}} = 1.3Q_{in}$, the external sludge recycle flow rate is $Q_{sl} = 0.5Q_{in}$, the wastage flow rate is $Q_w = 25.8\,l/h$ and the external carbon flow rate is $Q_{ext} = 0.0\,l/h$. In this case, an external carbon source is available, constituted by pure methanol, in a 33%-solution with a concentration of 80,000 mg COD/l. In the aerobic zones, the dissolved oxygen (DO) concentration is controlled in 2.0 mg O_2/l by simple PI controllers and in the anoxic zone it is assumed zero DO concentration.

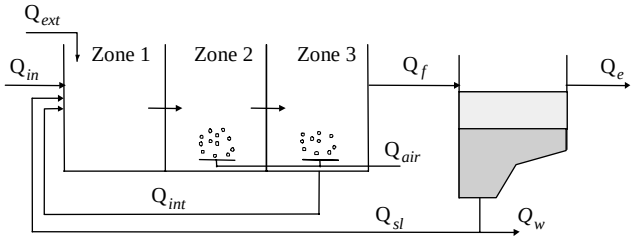

Fig. 1. Layout of the ASWWTP-USP benchmark.

For a reliable simulation of an ASP, the ASWWTP-USP benchmark is based on models widely accepted by the international community. Each bioreactor zone is modeled by IWA Activated Sludge Model ASM1 (Henze, Gujer, Marais, & Matsuo, 1987) and the secondary settler is modeled by the double-exponential settling velocity model of Takács, Patry, and Nolasco (1991). The complete plant model includes 52 large, complex, coupled nonlinear differential equations, which were implemented in Matlab/Simulink v.5.3. The values of the process parameters are here omitted, but they can be found in Sotomayor et al. (2001a). For more realistic simulations, a white noise, with zero-mean and standard deviation 0.05, was added to the outputs produced by the benchmark.

4. Identification of a subspace model for the ASP

4.1. Generation and pre-treatment of data set

It is not very easy to select either the input or the output variables of the process. In this work, the nitrate concentrations in the anoxic zone $S_{NO,1}$ (mg N/l) and in the last aerobic zone $S_{NO,3}$ (mg N/l) are selected as outputs. The internal recirculation rate Q_{int} (m^3/h) and the external carbon dosage Q_{ext} (l/h) are considered as inputs. However, to improve the model influent flow rate Q_{in} (m^3/h), influent readily biodegradable substrate $S_{S,in}$ (mg COD/l) and influent ammonium concentration $S_{NH,in}$ (mg N/l) are assumed as measurable disturbances, while influent nitrate concentration $S_{NO,in}$ (mg N/l) is assumed as an unmeasurable disturbance. The signals used in the identification procedure are summarized in Fig. 2. Note that all disturbances are considered as inputs.

Pseudo-random binary sequences (PRBS) are widely used in the identification of linear systems. The advantages of the PRBS input include easy implementation and an autocorrelation function similar to white noise. However, since the PRBS consists of only two levels, the resulting data may not provide sufficient information to excite nonlinear dynamics. Additionally, a PRBS signal of a too large magnitude may bias the estimation of the linear kernel. Multi-level (m-level) sequences, in contrast, allows the user to highlight nonlinear system behavior while manipulating the harmonic contents of the signal, reducing the effect of nonlinearities in the resulting linear model (Godfrey, 1993). On the other hand, the ill-conditioning of probing inputs may lead to a substantial deterioration of performance of the subspace algorithms. This possible cause of ill-conditioning has to do with wide variations in the amplitude of the input spectrum and with frequency bands, where the spectrum is nearly zero causing "insufficient excitation" (Chiuso & Picci, 2000).

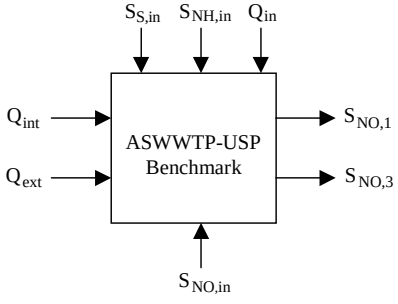

Fig. 2. Signals for subspace identification.

In the present paper, the input signals correspond to m-level uniformly distributed and not cross-correlated random sequences. These signals are periodic, deterministic and have autocorrelation functions similar to white noise. Their amplitudes and frequencies were chosen so as to adequately excite the system, without deviating too much from the normal operating point and, therefore, enabling the identification of a suitable linear model. All input and output data signals were acquired at a sampling rate of 9.6 min in a total period of 224 h (1400 samples).

For a better identification result, the raw data set is pre-processed. As the data set is generated from a simulation model, no data pre-filtering is necessary. However, since the system is running at an operating point different from zero and hence introducing some DC offsets, subtraction of the sample mean from the data set is done in order to remove these offsets. This operation is common in system identification (Söderström & Stoica, 1989). As pointed out by Chui (1997), it is important to make sure that the scales of the input–output data are of comparable sizes. Therefore, all data signals are normalized aiming to be equally weighted. Finally, the data set is detrended in order to remove linear trends from input–output data. This step is usual in signal processing. Asymptotic properties of subspace procedures, when the data set is pre-processed by removing trends and periodic components, are presented in (Bauer, 2000a). The pre-processed signals are shown in Figs. 3–5.

The identification procedure was carried out off-line in batch form by using the first 1000 samples of the data set, whereas the remaining 400 samples were applied to model validation. In the identification procedure the purely deterministic case was considered.

4.2. Order estimation

There is an extensive literature for order estimation algorithms related to linear, dynamical, state-space systems. Maybe the most important contribution can be attributed to the Akaike Information Criterion (*AIC*)

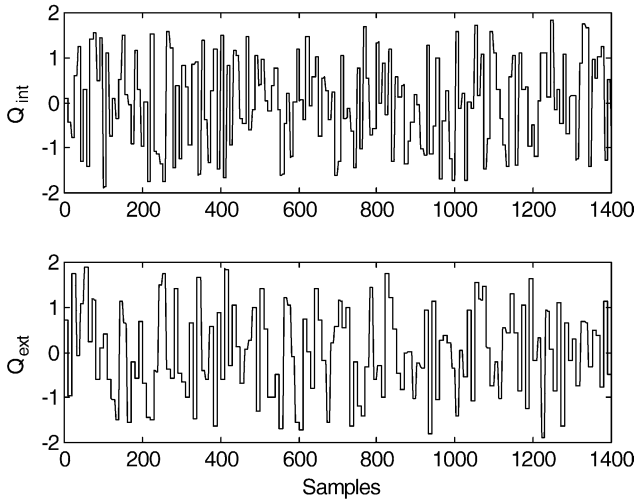

Fig. 3. Data sequences of the process: inputs.

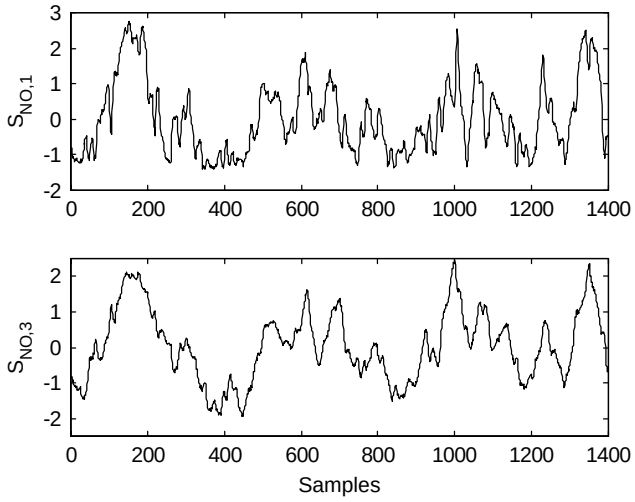

Fig. 5. Data sequences of the process: outputs.

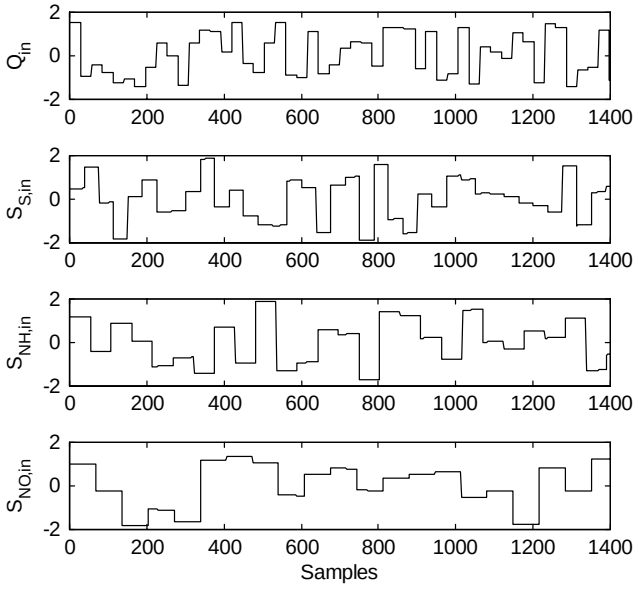

Fig. 4. Data sequences of the process: disturbances.

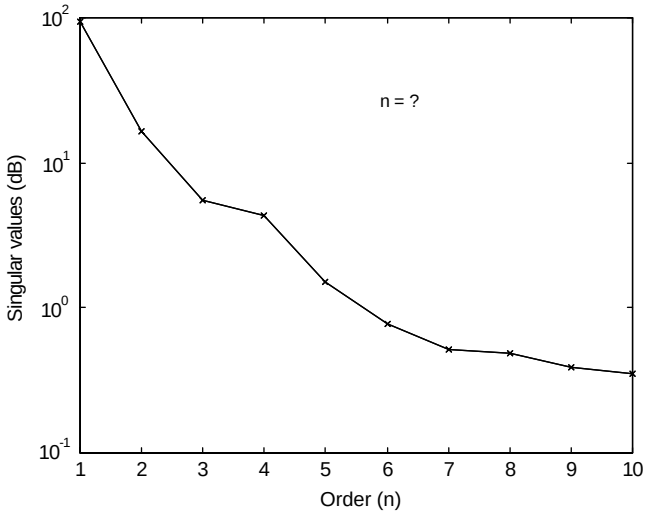

Fig. 6. Singular value spectrum.

for deciding the appropriate order of a statistical model (Akaike, 1973). Nevertheless, there are only a few references dealing with the order estimation in the context of subspace methods (Bauer, 2001).

In most of the subspace algorithms, the determination of the system order (n) is very subtle. Ideally, this information can be determined by the number of the nonzero singular values of the orthogonal (or oblique) projections of the row spaces of data block-Hankel matrices. However, in many situations, when the system data are noise corrupted, it is not so straightforward to calculate this number, so that the decision is taken by detecting a gap in the spectrum of the singular values. In this case, a visual inspection in Fig. 6 shows that the gap is not easy to determine, and hence the application of this strategy becomes subjective and the decision regarding the model order is an arbitrary one.

According to Bastogne, Noura, Sibille, and Richard (1998), a more practical procedure is to assume different system orders and then compare estimation errors, so that the value n is chosen in agreement with the minimum error. For instance, subspace identification methods do not involve error minimization schemes. These techniques are exclusive of the "classical" PEMs and they require a larger computational effort. The determination of the theoretical order, in the sense of minimization of the estimation error, is shown in Fig. 7, which was generated using the "robust" N4SID algorithm, where the grey-boxes represent the relative squared error (RSE) for $S_{NO,1}$, the white-boxes behind the grey-boxes, represent the RSE for $S_{NO,3}$. The solid-line represents the mean of the RSE indexes (*MRSE*). Comparing these relative estimation errors, it can be

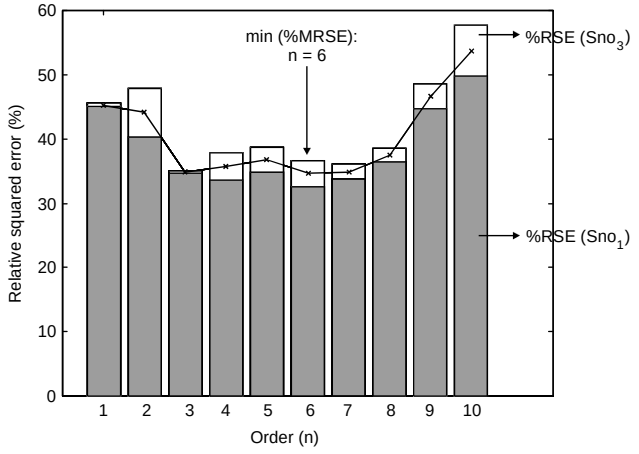

Fig. 7. Estimation error spectrum.

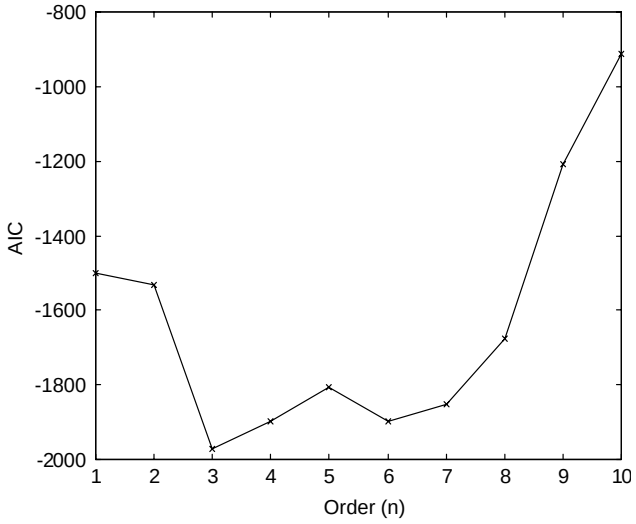

Fig. 8. AIC values spectrum.

noticed that the 3rd, 6th and 7th-order models have practically the same MRSE index, but for $n = 6$, it is slightly lower. Nevertheless, given that the 6th or 7th-order does not bring enough improvement in comparison with the 3rd-order model, the last one appears to be a reasonable choice. From the process control's point of view, the reduced 3rd-order model is quite suitable.

Anyway, for this particular case, the choice of n has not been totally convincing because the resulting model could not be accurate enough to represent the process, e.g., for monitoring purposes. Another alternative way to determine the system order is to apply the Akaike's statistical procedure (AIC), which is used by the CVA subspace method. The AIC is defined as

$$AIC(n) = N \ln\left[\sigma^2_{error}(n)\right] + 4M_n f_N, \tag{3}$$

where N is the number of samples, $\sigma^2_{error}(n)$ is the modeling error variance, using a n-order model with M_n parameters, and f_N is the small sample correction factor. The first term of the right-hand side in (3) quantifies the reduction in the modeling error variance due to the increase in the model order, whereas the second term penalizes this same increase. The model order n is selected corresponding to the minimum $AIC(n)$ value, which is considered as being optimal. Nevertheless, it must be emphasized that this criterion is essentially statistical and it does not necessarily assure that the optimum obtained model to be a valid one.

The number of parameters in the state-space model, deterministic system in (1), is fixed by the general state-space canonical form as in (Bauer, 2001):

$$M_n = nm + nl + lm \tag{4}$$

and the small sample correction factor is (Larimore, 1999):

$$f_N = \frac{N}{N - (M_n/l + (l+1)/2)}. \tag{5}$$

For a large N, $f_N \to 1$, which is the value used in the original AIC. In the present problem, the AIC is simply the mean of the $AICs$ values for each output of the n-order model. As shown in Fig. 8, the minimum AIC occurs for $n = 3$. Therefore, based on the obtained results, the suitable system order is 3, that correspond to the number of states or poles of the model.

4.3. Performance quality criteria

In Favoreel et al. (1999), three subspace algorithms (CVA, MOESP and "robust" N4SID) were applied to 15 different data sets from real-life systems. They evaluated the algorithms according to their computational complexity and prediction/simulation error and concluded that their performance is very similar. In the present paper, two performance indicators are proposed to measure identification/validation error, in order to obtain the best 3rd-order state-space model. The performance indicators are:

Mean relative squared error ($MRSE$):

$$\%MRSE = \frac{1}{l}\sum_{i=1}^{l}\sqrt{\frac{\sum_{j=1}^{N}(y_i(j) - \hat{y}_i(j))^2}{\sum_{j=1}^{N}(y_i(j))^2}} \times 100. \tag{6}$$

Mean variance-accounted-for ($MVAF$):

$$\%MVAF = \frac{1}{l}\sum_{i=1}^{l}\left(1 - \frac{\text{variance}(y_i - \hat{y}_i)}{\text{variance}(y_i)}\right) \times 100 \tag{7}$$

being y_i the ith real output and $\hat{y}_i$ the ith simulated output produced by the model. The $MRSE$ index is widely used in the literature, while the $MVAF$ index is, specifically, used by the SMI Toolbox. Both performance indexes are used to evaluate the adequacy of the model produced by each algorithm.

Table 1
Numerical results of the performance of the subspace-based algorithms

Algorithm	%MRSE		%MVAF	
	Identification	Validation	Identification	Validation
uCCA	40.4417	69.9404	83.5750	73.5628
cCCA	40.1652	69.2404	83.7998	73.9129
MOESP	31.8091	57.5806	89.9037	79.3096
N4SID	44.4914	72.9242	80.0546	74.2431
"robust" N4SID	34.8394	57.7508	87.8739	81.2475
DSR	34.2450	50.9904	88.2237	84.4274

Analyzing the values in Table 1, the MOESP model seems to produce a better model in terms of identification, while the DSR model seems to produce a better model in terms of validation. Hence, in this work, the 3rd-order DSR model was chosen to describe the process.

4.4. Identification results

The selected deterministic DSR model (proper model) is described by the following matrices:

$$A = \begin{bmatrix} 0.9763 & 0.0194 & 0.3268 \\ 0.0061 & 0.8815 & 0.0893 \\ -0.0023 & 0.0071 & 0.9763 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0238 & -0.0459 & -0.1488 & -0.0403 & 0.0002 \\ -0.1295 & 0.0299 & 0.0230 & 0.0185 & -0.0052 \\ 0.0097 & -0.0082 & -0.0082 & 0.0004 & 0.0036 \end{bmatrix} \quad (8)$$

$$C = \begin{bmatrix} 0.2253 & -0.4032 & -0.1823 \\ 0.2668 & 0.2880 & -0.4626 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.1292 & -0.0193 & -0.0651 & -0.0312 & 0.0053 \\ -0.0387 & 0.0086 & 0.0126 & 0.0105 & -0.0026 \end{bmatrix}$$

A strictly proper DSR model (i.e., with $D = 0$) is also identified, and it is described by

$$A = \begin{bmatrix} 0.9763 & 0.0199 & 0.3263 \\ 0.0062 & 0.8818 & 0.0907 \\ -0.0024 & 0.0072 & 0.9758 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0368 & -0.0434 & -0.1537 & -0.0431 & -0.0045 \\ -0.1505 & 0.0234 & 0.0357 & 0.0283 & -0.0044 \\ 0.0167 & -0.0100 & -0.0091 & 0.0003 & 0.0039 \end{bmatrix} \quad (9)$$

$$C = \begin{bmatrix} 0.2259 & -0.4026 & -0.1810 \\ 0.2664 & 0.2876 & -0.4633 \end{bmatrix}$$

The poles (eigenvalues of A) of the proper model (denoted by $+$) and the poles of the strictly proper model (denoted by Δ) are shown in Fig. 9. The poles closer to the unit circle are related to the slower system dynamics. The poles close to 1, show that the data set seems to contain a phenomenon known as "co-integration" in econometrics. Based on this observation, it is possible to obtain models which produce a one-step-ahead prediction error much smaller (Bauer, 2000b).

Figs. 10 and 11 show the outputs generated by the identified strictly proper model (dotted-line). As it can be observed, the identified model for a given operating

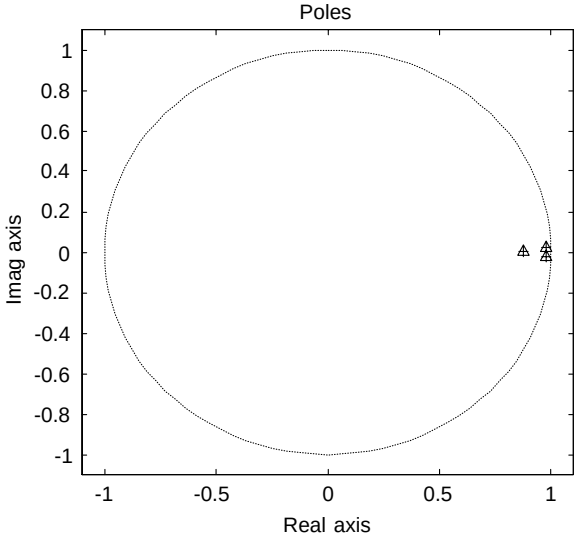

Fig. 9. Location of the poles of the DSR-models.

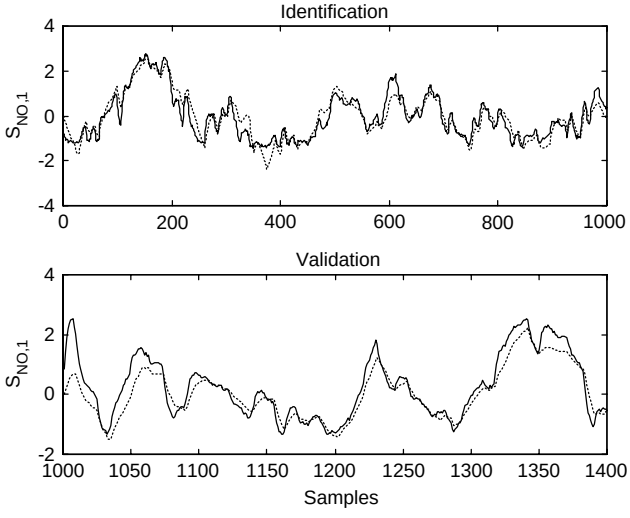

Fig. 10. Response comparison for $S_{NO,1}$.

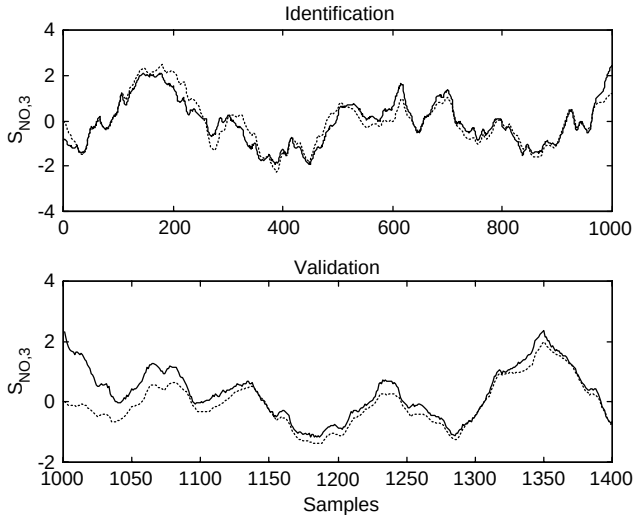

Fig. 11. Response comparison for $S_{NO,3}$.

point correctly reproduces the main dynamic characteristics of the ASP. In these graphics, either the identification or the validation data were introduced in the obtained model. In both cases the simulation started at zero initial conditions.

Given that low-order state-space models sufficiently representative of the nominal system behavior are a prerequisite to the systematic design of control systems, the strictly proper model (9) derived above has been successfully used in the implementation of an infinite-horizon LQ control (Sotomayor, Park, & Garcia, 2001b) and a model predictive control (Sotomayor & Garcia, 2002).

5. Conclusions

In this paper six subspace algorithms, namely uCCA, cCCA, MOESP, N4SID, "robust" N4SID and DSR, were used to identify a LTI discrete-time MIMO state-space model for a complex ASP, and their performance was compared. As a result, it is stated that a linear approach of such a nonlinear process is not impossible.

Three structural identification schemes, i.e. singular value inspection, estimation error minimization and the optimal statistical Akaike's criterion, were necessary to determine a very reduced order model ($n = 3$). The performance comparison study was focused on identification/validation accuracy issues of each subspace algorithm, through the use of two performance quality criteria. For this particular case, in general the results showed that the performance of the DSR algorithm was better than that of the "robust" N4SID, but that it was worse than that of the MOESP algorithm in producing a 3rd-order model in the identification context (with N large). On the other hand, in the validation context (with N small), the performance of the "robust" N4SID algorithm was better than that of the MOESP algorithm, but that it was worse than that of the DSR algorithm in producing a 3rd-order model. For the other algorithms, their performance was worse. The selected deterministic 3rd-order DSR model is able to describe reasonably well the process and, therefore, it is well suited for model-based control and monitoring applications.

It would be interesting to apply recent subspace identification methods for MIMO bilinear systems to this process, in order to study the improvement of the identified model quality and its effect on control and monitoring performances.

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