Master study
Systems and Control Engineering
Department of Technology
Telemark University College
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Topic: System identification and optimal estimation

Exercise 8, State estimation of quadruple tank process

Consider the quadruple tank process, Johansson (2000), with the non-linear state space model derived from mass balances and Bernulli's/Torricelli's law. The model may be developed as follows. Hence we have the following mass-balance equations

$$\begin{aligned} A_1\dot{x}_1 &= -q_1^{\text{out}} + q_3^{\text{out}} + q_1^{\text{inn}}, & (1)\\ A_2\dot{x}_2 &= -q_2^{\text{out}} + q_4^{\text{out}} + q_2^{\text{inn}}, & (2)\\ A_3\dot{x}_3 &= -q_3^{\text{out}} + q_3^{\text{inn}}, & (3)\\ A_4\dot{x}_4 &= -q_4^{\text{out}} + q_4^{\text{inn}}. & (4) \end{aligned}$$

The flow q^{out} out of a tank may be modeled using Bernulli's/Torricelli's law. By equating the potential energy and kinetic energy, i.e. $mgh = \frac{1}{2}mv^2$ and solving for the velocity we obtain $v = \sqrt{2gh}$. Multiplying with the area, a, of the outlet hole of the tank we obtain the volumetric flow-rate, q, out of the tank as $q = av = a\sqrt{2gh} = c\sqrt{h}$, i.e., the flow is proportional with the square root of the hight where $c = a\sqrt{2g}$.
Hence we then have that the flow out of the i'the tank is given by

$$q_i^{\text{out}} = a_i v_i = a_i\sqrt{2gh_i}. \tag{5}$$

The flow $q_1 = k_1u_1$ from pump 1 may be divided into a flow $q_1^{\text{inn}} = \gamma_1 k_1 u_1$ into tank 1 and a flow $q_4^{\text{inn}} = (1-\gamma_1)k_1u_1$ to tank 4, i.e. such that the flow from pump number 1 is $k_1u_1 = \gamma_1k_1u_1 + (1-\gamma_1)k_1u_1$. Here, γ_1 is a valve parameter which may be fixed such that $0 < \gamma_1 < 1$.
Similarly, the flow $q_2 = k_2u_2$ from the second pump may be divided into a flow $q_2^{\text{inn}} = \gamma_2k_2u_2$ into tank 2 and a flow $q_3^{\text{inn}} = (1-\gamma_2)k_2u_2$ into tank 3.
Here $0 < \gamma_1 < 1$ and $0 < \gamma_2 < 1$ are fixed valve parameters.
The system is non-minimum phase when choosing these parameters such that, $0 < \gamma_1 + \gamma_2 < 1$, and the system is minimum phase when, $1 < \gamma_1 + \gamma_2 < 2$.
Hence, a mass balance of the four tank process gives the state space model

$$\begin{aligned} A_1\dot{x}_1 &= -a_1\sqrt{2gx_1} + a_3\sqrt{2gx_3} + \gamma_1k_1u_1, & (6)\\ A_2\dot{x}_2 &= -a_2\sqrt{2gx_2} + a_4\sqrt{2gx_4} + \gamma_2k_2u_2, & (7)\\ A_3\dot{x}_3 &= -a_3\sqrt{2gx_3} + (1-\gamma_2)k_2u_2, & (8)\\ A_4\dot{x}_4 &= -a_4\sqrt{2gx_4} + (1-\gamma_1)k_1u_1, & (9) \end{aligned}$$

where $A_i \ \forall \ i = 1, \ldots, 4$ is the cross-section area of tank i, $a_i \ \forall \ i = 1, \ldots, 4$ is the cross-section area of the outlet pipe of tank i.
The numerical values for the above parameters, as well as nominal values for the states and control inputs, are chosen as presented in [**?**].

Exercise text

Part 1: Kalman filter

a) Simulate the non-linear model with fixed pump inputs u_1 and u_2.

c) Implement an Extended Kalman Filter (EKF) in parallel with the simulation of the non-linear model. Use a constant Kalman filter gain matrix.

Part 2: System identification of the Kalman filter

Real data input and output data from the quadruple tank process are available in the files, **Y_4tank.txt** and **U_4tank.txt**.

a) Use the observed real data from the quadruple tank process, Y, and U data files. Investigate the data. Use the D-SR Toolbox for MATLAB and identify a state space Kalman filter for the system.

A Quadruple tank model parameters

```
function [h10,h20,h30,h40,u10,u20,k1,k2,g1,g2]=param_4tank_nominal(izero)
% The nominal variables and parameters for the 4 tank process

if izero==1 % Minimum phase case
    h10=12.4; h20=12.7;
    h30=1.8; h40=1.4;
    u10=3.0; u20=3.0;
    k1=3.33; k2=3.35;
    g1=0.7; g2=0.6;
elseif izero==2
    h10=12.6; h20=13.0;
    h30=4.8; h40=4.9;
    u10=3.15; u20=3.15;
    k1=3.14; k2=3.29;
    g1=0.43; g2=0.34;
    %g1=0.2; g2=0.2;
    %g1=0.0; g2=0.0;
end
```

B Linearized continuous model matrices

```
function [A,B,D]=linmod_4tank(izero)
% The 4 tank linearized model matrices

[A1,A2,A3,A4,a1,a2,a3,a4,kc,g]=param_4tank_model;

[h10,h20,h30,h40,u10,u20,k1,k2,g1,g2]=param_4tank_nominal(izero);

T1=(A1/a1)*sqrt(2*h10/g);
T2=A2*sqrt(2*h20/g)/a2;
T3=A3*sqrt(2*h30/g)/a3;
T4=A4*sqrt(2*h40/g)/a4;

A=[-1/T1,0    ,A3/(A1*T3),0
    0   ,-1/T2,0         ,A4/(A2*T4)
    0   ,0    ,-1/T3     ,0
    0   ,0    ,0         ,-1/T4];

B=[g1*k1/A1     ,0
    0           ,g2*k2/A2
    0           ,(1-g2)*k2/A3
    (1-g1)*k1/A4,0];

D=[kc, 0,0,0
    0,kc,0,0];
```

C Quadruple tank model parameters

```
function [A1,A2,A3,A4,a1,a2,a3,a4,kc,g]=param_4tank_model
% Parameters for the 4 tank level process

A1=28; A3=28;
A2=32; A4=32;
a1=0.071; a3=0.071;
a2=0.057; a4=0.057;
kc=0.50;
g=981;
```

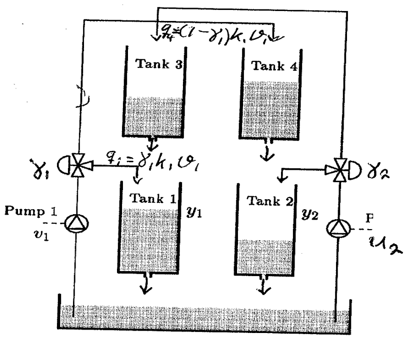

Figure 1: Figureof the quadruple tank process.