Mandatory Exercise 1

IIA2217 System Identification and Optimal Estimation

Solution Proposal

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Task (20%): Diverse Questions

Answer the following:

a) *System Identification may be defined as the field of building mathematical models of dynamic systems based on observed and measured input and output data from the system. System Identification was defined by Lotfi Zadeh (1962) as: Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent.*

b) *The Kalman filter is an an optimal minimum variance state estimator, i.e. the Kalman filter is optimal in a minimum variance sense. Hence, assuming x the true state and $\hat{x}$ the Kalman filter state estimate of x then the covariance matrix $\hat{X} = E((x - \hat{x})(z - \hat{x})^T)$ is minimized*

c) *A linear deterministic state space model is*

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$
$$y_k = Dx_k + Eu_k \tag{2}$$

where $k \ \forall \ k = 0, 1, \ldots,$ is discrete time and x_0 the initial state.

the steady state gain is obtained by letting time go to infinity, assuming a stable system, and $x_{k+1} = x_k = x$ which gives the gain H_d, i.e.

$$y_k = H_d u_k \tag{3}$$
$$H_d = D(I - A)^{-1}B + E \tag{4}$$

The impulse response matrices are given by $H_1 = DB$, $H_2 = DAB$, $H_3 = DA^2B$, ..., i.e.

$$H_i = DA^{i-1}B \ \forall \ k = 1, 2, 3, \ldots \tag{5}$$

d) *Here we have three equations and two unknown parameters a and b. We may write the equations as a linear regression model $Y = XB$ where*

$$\overbrace{\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}}^{Y} = \overbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}^{X} \overbrace{\begin{bmatrix} a \\ b \end{bmatrix}}^{B} \tag{6}$$

The Ordinary Least Squares estimate B_{OLS} of B minimizing the Frobenius norm $||Y - XB||_F$ i given by

$$B_{OLS} = (X^T X)^{-1} X^T Y = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \qquad (7)$$

e)