Mandatory Exercise 1 IIA2217 System Identification and Optimal Estimation

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Task: Diverse Questions

Answer the following:

- a) Define the term System Identification ?
- b) Give an example of an : Optimal state estimator. Why is the estimator optimal?
- c) Consider a discrete time system with input u_k and output y_k where $k = 0, 1, \ldots$ is discrete time. Answer the following:
 - Propose a linear dynamic state space model for the system.
 - Define the impulse response matrices ?
- d) Given three equations 1 = a + b, 4 = 2a + b and 5 = 3a + b and two unknown parameters a and b.
 - Is it possible to calculate a and b?
 - If so, find estimates of a and b !
 - Comment upon the solution.

Tips: Formulate the equations into the linear regression model Y = XBwhere the coefficients a and b are stacked in a vector B. We have

$$\overbrace{\left[\begin{array}{c}1\\4\\5\end{array}\right]}^{Y} = \overbrace{\left[\begin{array}{c}1&1\\2&1\\3&1\end{array}\right]}^{X} \overbrace{\left[\begin{array}{c}a\\b\end{array}\right]}^{B} = \overbrace{\left[\begin{array}{c}1&1\\b\end{array}\right]}^{B} (1)$$

e)

Some useful theory about orthogonal projections is defined in the following Lemma !

Lemma 0.1

Consider given a linear matrix equation

$$Y = \Theta Z \tag{2}$$

where Y and Z are two known matrices of appropriate dimensions.

Then, the following projection holds

$$Y/Z = Y, (3)$$

where the / (slash) projection operator is defined as

$$Y/Z = YZ^T (ZZ^T)^* Z, (4)$$

where $(ZZ^T)^*$ is the pseudo inverse of matrix ZZ^T . Furthermore $(ZZ^T)^* = (ZZ^T)^{-1}$ if the indicated inverse exists.

Proof 0.1 (Proof of Lemma 0.1) We have

$$Y/Z = \Theta \overleftarrow{Z/Z} = \Theta Z = Y.$$
(5)

Questions:

- Find the ordinary Least squares (OLS) estimate of Θ ?
- What is the OLS prediction \overline{Y} of Y?
- Sketch an (x,y) diagram of the projections ? Tips: Similar as in Ch4.5 lecture notes.
- **f**) Given a general system described by the combined deterministic and stochastic system state space model on innovations form model

$$x_{k+1} = Ax_k + Bu_k + Ce_k, (6)$$

$$y_k = Dx_k + Eu_k + Fe_k, (7)$$

where x_k is the *n* dimensional predicted state vector. Assume a known sequence of *N* input and output observations

$$(u_k, y_k) \ \forall \ k = 0, 1, \dots, N-1$$
 (8)

are known.

• How may we identify the system order n and the extended observability matrix O_L , as well as the model matrices A, D. Tips: Find the projection matrix $Z_{J|L} = O_L X_J^a$ where X_J^a is a projected predicted state vector, and use the Singular Value Decomposition (SVD) of the matrix $Z_{J|L}$ in order to identify n and the extended observability matrix O_L .