# Mandatory Exercise 1 IIA2217 System Identification and Optimal Estimation 

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## Task: Diverse Questions

Answer the following:
a) Define the term System Identification?
b) Give an example of an : Optimal state estimator. Why is the estimator optimal?
c) Consider a discrete time system with input $u_{k}$ and output $y_{k}$ where $k=$ $0,1, \ldots$ is discrete time. Answer the following:

- Propose a linear dynamic state space model for the system.
- Define the impulse response matrices ?
d) Given three equations $1=a+b, 4=2 a+b$ and $5=3 a+b$ and two unknown parameters $a$ and $b$.
- Is it possible to calculate $a$ and $b$ ?
- If so, find estimates of $a$ and $b$ !
- Comment upon the solution.

Tips: Formulate the equations into the linear regression model $Y=X B$ where the coefficients $a$ and $b$ are stacked in a vector B. We have

e)

Some useful theory about orthogonal projections is defined in the following Lemma!

## Lemma 0.1

Consider given a linear matrix equation

$$
\begin{equation*}
Y=\Theta Z \tag{2}
\end{equation*}
$$

where $Y$ and $Z$ are two known matrices of appropriate dimensions.

Then, the following projection holds

$$
\begin{equation*}
Y / Z=Y \tag{3}
\end{equation*}
$$

where the / (slash) projection operator is defined as

$$
\begin{equation*}
Y / Z=Y Z^{T}\left(Z Z^{T}\right)^{*} Z \tag{4}
\end{equation*}
$$

where $\left(Z Z^{T}\right)^{*}$ is the pseudo inverse of matrix $Z Z^{T}$. Furthermore $\left(Z Z^{T}\right)^{*}=$ $\left(Z Z^{T}\right)^{-1}$ if the indicated inverse exists.

Proof 0.1 (Proof of Lemma 0.1) We have

$$
\begin{equation*}
Y / Z=\Theta \overbrace{Z / Z}^{Z}=\Theta Z=Y . \tag{5}
\end{equation*}
$$

## Questions:

- Find the ordinary Least squares (OLS) estimate of $\Theta$ ?
- What is the OLS prediction $\bar{Y}$ of $Y$ ?
- Sketch an ( $\mathrm{x}, \mathrm{y}$ ) diagram of the projections ? Tips: Similar as in Ch4.5 lecture notes.
f) Given a general system described by the combined deterministic and stochastic system state space model on innovations form model

$$
\begin{align*}
x_{k+1} & =A x_{k}+B u_{k}+C e_{k},  \tag{6}\\
y_{k} & =D x_{k}+E u_{k}+F e_{k}, \tag{7}
\end{align*}
$$

where $x_{k}$ is the $n$ dimensional predicted state vector. Assume a known sequence of $N$ input and output observations

$$
\begin{equation*}
\left(u_{k}, y_{k}\right) \forall k=0,1, \ldots, N-1 \tag{8}
\end{equation*}
$$

are known.

- How may we identify the system order $n$ and the extended observability matrix $O_{L}$, as well as the model matrices $A, D$. Tips: Find the projection matrix $Z_{J \mid L}=O_{L} X_{J}^{a}$ where $X_{J}^{a}$ is a projected predicted state vector, and use the Singular Value Decomposition (SVD) of the matrix $Z_{J \mid L}$ in order to identify $n$ and the extended observability matrix $O_{L}$.

