Mandatory Exercise 1

IIA2217 System Identification and Optimal Estimation

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Task: Diverse Questions

Answer the following:

a) Define the term **System Identification** ?

b) What is: **a Kalman filter**

c) Consider a discrete time system with input u_k ans outpot y_k where $k = 0, 1, \ldots$ is discrete time. Answer the following:

- Propose a linear state space model for the system.
- Define the system gain.
- Define the impulse response matrices ?

d) Given three equations $3 = a + b$, $4 = 2a + b$ and $5 = 3a + b$ and two unkown parameters a and b.

- Is it possible to calculate a and b ?
- If so, find estimates of a and b !
- Comment upon the solution.

Tips: Formulate the equations into the linear regression model $Y = XB$ where the coefficients a and b are stacked in

$$\overbrace{\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}}^{Y} = \overbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}^{X} \overbrace{\begin{bmatrix} a \\ b \end{bmatrix}}^{B} \tag{1}$$

e)

Some useful theory about orthogonal projections is defined in the following Lemma !

Lemma 0.1
Consider given a linear matrix equation

$$Y = \Theta Z \tag{2}$$

where Y and Z are two known matrices of appropriate dimensions. Then, the following projection holds

$$Y/Z = Y, \tag{3}$$

where the / (slash) projection operator is defined as

$$Y/Z = YZ^T(ZZ^T)^*Z, \tag{4}$$

where $(ZZ^T)^*$ is the pseudo inverse of matrix ZZ^T. Furthermore $(ZZ^T)^* = (ZZ^T)^{-1}$ if the indicated inverse exists.

Proof 0.1 (Proof of Lemma 0.1) *We have*

$$Y/Z = \Theta \overbrace{Z/Z}^{Z} = \Theta Z = Y. \tag{5}$$

Questions:

- Find the ordinary Least squares (OLS) estimate of Θ ?
- What is the OLS prediction $\bar{Y}$ of Y ?

f) Given an autonomous system described by the innovations form state space model

$$\begin{aligned} x_{k+1} &= Ax_k, && (6) \\ y_k &= Dx_k, && (7) \end{aligned}$$

where x_k is the n dimensional state vector.
Assume a known sequence of N output observations

$$y_k \ \forall \ k = 0, 1, \ldots, N-1 \tag{8}$$

are known.

- How may we identify the model matrices A, D and the initial state vector x_0, including the system order n ?
- Tips: Task 1 in the note with exercises/tasks !