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DDiR, January 12, 2021

IIA2217 System Identification and Optimal Estimation

1 Finite Impulse Response (FIR) models

1.1 Theory on Finite Impulse Response (FIR) models

Consider a standard discrete time linear state space model as follows

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k, \quad (1) \\ y_k &= Dx_k, \quad (2) \end{aligned}$$

and the discrete transfer function model equivalent

$$\begin{aligned} qx_k &= Ax_k + Bu_k, \quad (3) \\ y_k &= Dx_k, \quad (4) \end{aligned}$$

i.e., such that q is the shift operator such that $qx_k = x_{k+1}$ and $q^{-1}x_k = x_{k-1}$. Hence, we have the transfer function input-output polynomial model

$$y_k = H(q)u_k, \quad (5)$$

where the transfer matrix can be expressed as

$$\begin{aligned} H(q) &= D(qI - A)^{-1}B \\ &= \sum_{i=1}^{\infty} DA^{i-1}Bq^{-i} = DBq^{-1} + DABq^{-2} + DA^2Bq^{-3} + \cdots \quad (6) \end{aligned}$$

This gives rise to a truncated input and output FIR model as follows

$$\begin{aligned} y_k &= h_1u_{k-1} + h_2u_{k-2} + h_3u_{k-3} + \cdots + h_Mu_{k-M} \\ &= DC_Mu_{k-M|M}, \quad (7) \end{aligned}$$

where

$$h_i = DA^{i-1}B \ \forall \ i = 1, 2, \ldots, M \quad (8)$$

are the impulse responses of the system and M is the number of terms in the FIR model. M is also defined as the model horizon.

Here we have defined the *reversed extended controllability* matrix, C_M^d, for the pair (A, B) is defined as

$$C_M^d \stackrel{\text{def}}{=} \begin{bmatrix} A^{M-1}B & A^{M-2}B & \cdots & B \end{bmatrix} \in \mathbb{R}^{n\times ir}, \tag{9}$$

where the subscript $i = M$ denotes the number of block columns.

By putting $k := k + M$ in Eq. (7) we obtain the linear regression model

$$\begin{aligned} y_{k+M} &= h_1 u_{k+M-1} + h_2 u_{k+M-2} + h_3 u_{k+M-3} + \cdots + h_M u_k \\ &= DC_M u_{k|M}, \end{aligned} \tag{10}$$

Assume $N > 1$ known input and output data vectors, i.e., u_k and y_k for the time instants $\forall\ k = 1, 2, \ldots, N$.

Using the time instants $k = 1, 2, \ldots, N - M$ in Eq. (10) gives the linear regression model

$$Y_{M+1|1} = \overbrace{DC_M}^{\Theta} U_{1|M}, \tag{11}$$

where

$$Y_{M+1|1} = \begin{bmatrix} y_{M+1} & y_{M+2} & \cdots & y_N \end{bmatrix} \in \mathbb{R}^{m\times(N-M)}, \tag{12}$$

$$U_{1|M} = \begin{bmatrix} u_{1|M} & u_{2|M} & \cdots & u_{N-M|M} \end{bmatrix} \in \mathbb{R}^{rM\times(N-M)}. \tag{13}$$

Example 1.1 *Given $N = 10$ and $M = 3$. We then have the following linear regression model*

$$Y_{4|1} = \Theta U_{1|3}, \tag{14}$$

where the parameter matrix $\Theta = DC_M$ of impulse response matrices are given by

$$\Theta = DC_3 = \begin{bmatrix} DA^2B & DAB & DB \end{bmatrix} = \begin{bmatrix} h_3 & h_2 & h_1 \end{bmatrix}. \tag{15}$$

The data matrices $Y_{4|1}$ and $U_{1|3}$ are given by

$$Y_{4|1} = \begin{bmatrix} y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_{10} \end{bmatrix} \in \mathbb{R}^{m\times(N-M)}, \tag{16}$$

and

$$U_{1|3} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\ u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \\ u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 \end{bmatrix} \in \mathbb{R}^{rM\times(N-M)}, \tag{17}$$

As we se, all relevant data are used in the linear regression model Eq. (14) with data matrices as in Eqs. (16)-(17). We also se that input u_{10} are not used. The reason is that in a dynamic model as in Eqs. (1) and (2) the input at time instant k are influencing the output at time k+1, and output y_{11} are not used. In order to also use the input u_k at time instant $k = 10$ the direct feed-through term $y_k = Eu_k$ should be included in the regression model but it is not an option here.