Mandatory Exercise 1
IIA2217 System Identification and Optimal Estimation

David Di Ruscio

January 18, 2024

Task: Diverse Questions

Answer the following:

a) Define the term **System Identification** ?

b) Give an example of an : **Optimal state estimator.** Why is the estimator optimal?

c) Consider a discrete time system with input u_k and output y_k where $k = 0, 1, \ldots$ is discrete time. Answer the following:

- Propose a linear dynamic state space model for the system.
- Define the impulse response matrices ?

d) Given three equations $1 = a + b$, $4 = 2a + b$ and $5 = 3a + b$ and two unknown parameters a and b.

- Is it possible to calculate a and b ?
- If so, find estimates of a and b !
- Comment upon the solution.

Tips: Formulate the equations into the linear regression model $Y = XB$ where the coefficients a and b are stacked in a vector B. We have

$$\overbrace{\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}}^{Y} = \overbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}^{X} \overbrace{\begin{bmatrix} a \\ b \end{bmatrix}}^{B} \tag{1}$$

e)

Some useful theory about orthogonal projections is defined in the following Lemma !

Lemma 0.1
Consider given a linear matrix equation

$$Y = \Theta Z \tag{2}$$

where Y and Z are two known matrices of appropriate dimensions.

Then, the following projection holds

$$Y/Z = Y, \tag{3}$$

where the / (slash) projection operator is defined as

$$Y/Z = YZ^T(ZZ^T)^*Z, \tag{4}$$

where $(ZZ^T)^*$ *is the pseudo inverse of matrix* ZZ^T. *Furthermore* $(ZZ^T)^* = (ZZ^T)^{-1}$ *if the indicated inverse exists.*

Proof 0.1 (Proof of Lemma 0.1) *We have*

$$Y/Z = \Theta \overbrace{Z/Z}^{Z} = \Theta Z = Y. \tag{5}$$

Questions:

- Find the ordinary Least squares (OLS) estimate of Θ ?
- What is the OLS prediction $\bar{Y}$ of Y ?
- Sketch an (x,y) diagram of the projections ? Tips: Similar as in Ch4.5 lecture notes.

f) Given a general system described by the combined deterministic and stochastic system state space model on innovations form model

$$x_{k+1} = Ax_k + Bu_k + Ce_k, \tag{6}$$

$$y_k = Dx_k + Eu_k + Fe_k, \tag{7}$$

where x_k is the n dimensional predicted state vector.
Assume a known sequence of N input and output observations

$$(u_k, y_k) \;\forall\; k = 0, 1, \ldots, N-1 \tag{8}$$

are known.

- How may we identify the system order n and the extended observability matrix O_L, as well as the model matrices A, D. Tips: Find the projection matrix $Z_{J|L} = O_L X_J^a$ where X_J^a is a projected predicted state vector, and use the Singular Value Decomposition (SVD) of the matrix $Z_{J|L}$ in order to identify n and the extended observability matrix O_L.