Final Exam SCE2202
System identification and optimal estimation
Monday 4. June 2007 Time: kl. 9.00 - 12.00

The final exam consists of: 4 tasks.
The exam counts 70% of the final grade.
Available aids: pen and paper

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Task 1 (20%): Prediction error methods

Task 2 (20%): Ordinary Least Squares method

a) From the state space model we have the equations

$$\begin{aligned} x^1_{k+1} &= x^2_k + b_1 u_k && (1) \\ x^2_{k+1} &= a_1 x^1_k + a_2 x^2_k + b_2 u_k && (2) \\ y_k &= x^1_k && (3) \end{aligned}$$

From (1) we have

$$x^2_k = x^1_{k+1} - b_1 u_k \tag{4}$$

$$\tag{5}$$

Substituting (4) into (2) gives

$$x^1_{k+2} - b_1 u_{k+1} = a_1 x^1_k + a_2(x^1_{k+1} - b_1 u_k) + b_2 u_k \tag{6}$$

Rearranging and using (3) in (6) gives

$$y_{k+2} = a_2 y_{k+1} + a_1 y_k + b_1 u_{k+1} + (b_2 - a_2 b_1) u_k \tag{7}$$

Putting $k := k - 2$ gives

$$\begin{aligned} y_k &= a_2 y_{k-1} + a_1 y_{k-2} + b_1 u_{k-1} + (b_2 - a_2 b_1) u_{k-2} && (8) \\ &= \overbrace{\begin{bmatrix} y_{k-1} & y_{k-2} & u_{k-1} & u_{k-2} \end{bmatrix}}^{\varphi_k^T} \overbrace{\begin{bmatrix} a_2 \\ a_1 \\ b_1 \\ b_2 - a_2 b_1 \end{bmatrix}}^{\theta_0} && (9) \end{aligned}$$

Task 3 (15%): Prediction error methods

a) This is an Auto Regression Moving average with eXtra inputs (ARMAX) model.

b) An ARX model can be written in the following polynomial form

$$A(q)y_k = B(q)u_k + e_k \tag{10}$$

Hence, with $C(q) = I$.

c) An Output Error (OE) model can be written in the following polynomial form

$$A(q)y_k = B(q)u_k + A(q)e_k \tag{11}$$

Hence, with $C(q) = A(q)$.

d) We have the input and output model, obtained by eliminating the state x_k from the model

$$y_k - ay_{k-1} = 2u_k + (b - 2a)u_{k-1} + e_k + (k - a)e_{k-1} \tag{12}$$

This can be written as an ARMAX model with polynomials

$$\begin{aligned} A(q) &= 1 - aq^{-1} && (13) \\ B(q) &= 2 + (b - 2a)q^{-1} && (14) \\ C(q) &= 1 + (k - a)q^{-1} && (15) \end{aligned}$$

- This become an ARX model if $k = a$.
- This become an OE model if $k = 0$.

Task 4 (15%): Prediction error methods

a) The FIR model with known data can be written in matrix form as follows

$$\overbrace{\begin{bmatrix} y_5^T \\ y_6^T \\ \vdots \\ y_N^T \end{bmatrix}}^{Y} = \overbrace{\begin{bmatrix} u_4^T & u_3^T & u_2^T & u_1^T \\ u_5^T & u_4^T & u_3^T & u_2^T \\ \vdots & \vdots & \vdots & \vdots \\ u_{N-1}^T & u_{N-2}^T & u_{N-3}^T & u_{N-4}^T \end{bmatrix}}^{X} \overbrace{\begin{bmatrix} h_1^T \\ h_2^T \\ h_3^T \\ h_4^T \end{bmatrix}}^{B} + \overbrace{\begin{bmatrix} w_5^T \\ w_6^T \\ \vdots \\ w_N^T \end{bmatrix}}^{E} \tag{16}$$

The impulse response matrices may be identified by the OLS solution

$$B = (X^T X)^{-1} X^T Y \tag{17}$$