Partial test, Course SCE2206 System Identification and Optimal estimation Friday 2. March 2007 Time: kl. 11.15 - 13.15

The partial test consists of: 2 tasks.
The test counts 30% of the final grade in the course.
No aids available except pen and paper

Contact for the test: David Di Ruscio
Systems and Control Engineering
Department of technology
Telemark University College
N-3914 Porsgrunn

Task 1 (22%): Subspace System Identification

Given a system which can be described by the following discrete time model on innovations form

$$x_{k+1} = Ax_k + Bu_k + Ce_k, \tag{1}$$
$$y_k = Dx_k + Eu_k + Fe_k \tag{2}$$

where e_k is white with zero mean and unit covariance matrix, i.e., $\mathrm{E}(e_k e_k^T) = I$ and where the following series of output and input data is collected and assumed to be known

$$Y = \begin{bmatrix} y_0^T \\ y_1^T \\ \vdots \\ y_{N-1} \end{bmatrix} \in \mathbb{R}^{N \times m}, \; U = \begin{bmatrix} u_0^T \\ u_1^T \\ \vdots \\ u_{N-1} \end{bmatrix} \in \mathbb{R}^{N \times r}. \tag{3}$$

a) Based on the model in Equations (1) and (2) and with known data as given in (3) we can develop the following matrix equations

$$Y_{J|L} = O_L X_J + H_L^d U_{J|L} + H_L^s E_{J|L}, \tag{4}$$
$$Y_{J+1|L} = \tilde{A}_L Y_{J|L} + \tilde{B}_L U_{J|L+1} + \tilde{C}_L E_{J|L+1}, \tag{5}$$

where J and L are user specified non-negative integer numbers.

1. Write down the structure of the matrices in the matrix equations, (4) and (5), with parameters $N = 8$, $L = 2$, $J = 2$ and $g = 1$.
2. Find a formula for the number of columns, K, in the data matrices which are involved in the matrix Equations (4) and (5). Investigate if the formula is consistent with the number of columns you get in Step 1 above.

b) With basis in the known data, (3), and the matrix Equation (4) it is possible to develop the matrix equation

$$Z_{J|L} = O_L X_J^a \tag{6}$$

where the data matrix $Z_{J|L}$ is known.

Furthermore, with the basis in the matrix equation (5) it is possible to develop the matrix equation

$$Z_{J+1|L} = \tilde{A}_L Z_{J|L} \tag{7}$$

where the data matrices $Z_{J+1|L}$ are $Z_{J|L}$ known.

Find expressions for the data matrices $Z_{J+1|L}$ and $Z_{J|L}$ for the following three cases:

- an autonomous system, i.e. when $u_k = 0$ and $e_k = 0$.
- a deterministic system, i.e., when $e_k = 0$.
- a stochastic system, i.e., when $u_k = 0$.
- a general combined deterministic and stochastic system.

Remark: you are to define the projections which is involved in the expressions for $Z_{J+1|L}$ and $Z_{J|L}$. That is orthogonal projections of the type A/B and $B^{\perp}$ for two matrices A and B.

c) Show how

- the system order, n
- the extended observability matrix O_L
- the system matrices A and D

can be estimated.

d) From known data as given in (3) and the matrix equation (5), the following matrix a matrix equation can be developed

$$Z^d_{J+1|L} = \tilde{A}_L Z^d_{J|L} + \tilde{B}_L U_{J|L+1} \tag{8}$$

where the data matrices $Z^d_{J+1|L}$ and $Z^d_{J|L}$ are known.

1. Find expressions for the data matrices $Z^d_{J+1|L}$ and $Z^d_{J|L}$ in the matrix equations (8). Assume a general combined deterministic and stochastic system.
2. Explain how the matrices B and E can be estimated. Take with advantage a particular example with $L = 2$ as the starting point.

e) Consider now the following Kalman filter on innovations form

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + K\varepsilon_k, & (9) \\ y_k &= Dx_k + Eu_k + \varepsilon_k & (10) \end{aligned}$$

What is the relationship between the Kalman filter on innovations form in Equations (9) and (10) and the innovations formulation in Equations (1) and (2)?

f) Consider that the known input and output data as given in (3) are collected in closed loop, i.e., we assume that there is feedback in the known data. Consider the projection

$$
\begin{aligned}
Z^s_{J|1} &= FE_{J|1} = Y_{J|1} - Y_{J|1} / \begin{bmatrix} U_{0|J} \\ Y_{0|J} \end{bmatrix} \\
&= \begin{bmatrix} Fe_J & Fe_{J+1} & \dots & Fe_{N-1} \end{bmatrix} \\
&= \begin{bmatrix} \varepsilon_J & \varepsilon_{J+1} & \dots & \varepsilon_{N-1} \end{bmatrix}
\end{aligned} \tag{11}
$$

Explain how this projection can be used in order to develop a subspace identification algorithm for closed loop systems.

Task 2 (8%): Regression

a) Given a system modelled by the linear model

$$Y = XB + E \tag{12}$$

where $Y \in \mathbb{R}^{N \times m}$ and $X \in \mathbb{R}^{N \times r}$ are known data matrices $E \in \mathbb{R}^{N \times m}$ is a matrix of normally distributed white noise. We are assuming a large number of observations, N, such that $N > r$.

Find an Ordinary Least Squares (OLS) estimate, B_{OLS}, of the unknown matrix of regression coefficients, B.

b) Assume now that tha data matrix X is not of full column rank. Show how you can perform a principal Component Analysis (PCA) of the X matrix by using a Singular Value Decomposition (SVD). What is the number of Principal Components, a.

c) Find a Principal Component Regression (PCR) estimate, B_{PCR}, of the unknown matrix of regression coefficients, B.

d) Find a formula for the Partial Least Squares (PLS) estimate, B_{PLS}, of the unknown matrix of regression coefficients, B.