Lecture 12

Recursive OLS Ch. 7

1. Introduction

- Offline SID/PEM methods

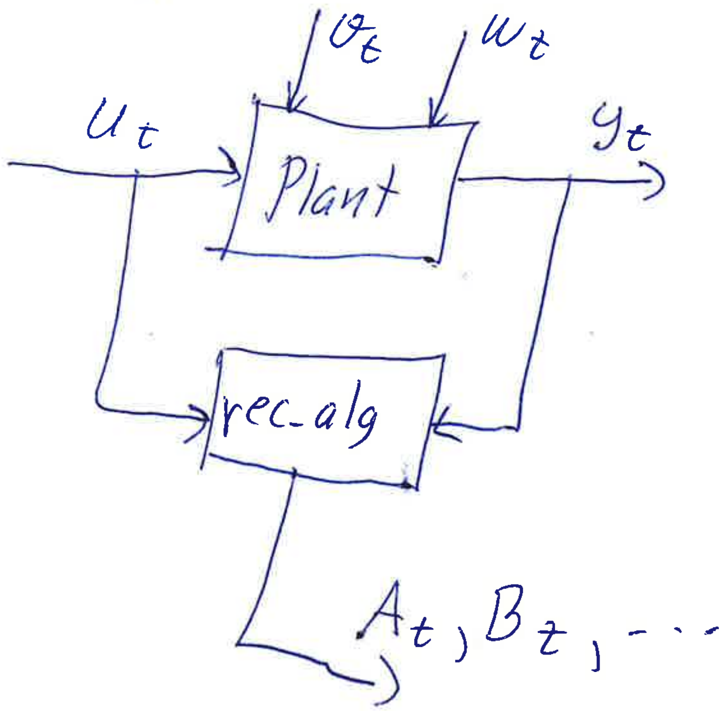
  model = alg(Y, U)

  Y and U data matrices from a sequence of N observations $\left.\begin{matrix} u_k \\ y_k \end{matrix}\right\} \forall k = 1, \dots, N$

- Recursive algorithms.
  - most offline methods have a recursive variant

$$mod_t = rec\_alg(y_t, u_t)$$

$$[A_t, B_t, D_t] = rec\_alg(y_t, u_t)$$



2 ROLS

- Ex. the mean
- given N observations $y_k \; \forall k=1,\dots,N$

$$\bar{y}_N = \frac{1}{N}\sum_{k=1}^{N} y_k$$

- Replace N with present time t. The mean at present time is then

$$\bar{y}_t = \frac{1}{t}\sum_{k=1}^{t} y_k$$

• Recursive alg. Divide the sum

$$\bar{y}_t = \frac{1}{t}\left(\sum_{k=1}^{t-1} y_k + y_t\right) = \frac{t-1}{t}\underbrace{\frac{1}{t-1}\sum_{k=1}^{t-1} y_k}_{\bar{y}_{t-1}} + \frac{1}{t}y_t$$

Hence

$$\bar{y}_t = \left(1-\frac{1}{t}\right)\bar{y}_{t-1} + \frac{1}{t}y_t = \bar{y}_{t-1} + \frac{1}{t}\left(y_t - \bar{y}_{t-1}\right)$$

Alg

Step 0 At startup $t=1$. Specify $\bar{y}_0$ $\bar{y}_1 = y_1$

Step 1 At all other time instants

$$\bar{y}_t = \bar{y}_{t-1} + \frac{1}{t}\left(y_t - \bar{y}_{t-1}\right)$$

- Offline OLS, Eq. (55)

$$\hat{\theta}_N = \left(\sum_{k=1}^{N} \varphi_k \Lambda \varphi_k^T\right)^{-1} \sum_{k=1}^{N} \varphi_k \Lambda y_k$$

where

$$y_k = \varphi_k^T \theta + e_k$$

the linear regression model

$y_k \in \mathbb{R}^m$, $\theta \in \mathbb{R}^p$, $[\varphi_k^T] \in \mathbb{R}^{m \times p}$

and $\varphi_k \in \mathbb{R}^{p \times m}$

$${}_{p}[\varphi_k]^{m} \; {}_{m}[\Lambda]^{m} \; {}_{m}[y_k]^{1} = {}_{p}[\varphi_k \Lambda y_k]^{1}$$

P_t

- Replace N with present time t

$$\hat{\theta}_t = \left(\sum_{k=1}^{t} \varphi_k \Lambda \varphi_k^T\right)^{-1} \sum_{k=1}^{t} \varphi_k \Lambda y_k \qquad (1)$$

We have two sums.

- Define matrix

$$P_t = \left(\sum_{k=1}^{t} \varphi_k \Lambda \varphi_k^T\right)^{-1}$$

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which gives

$$P_t^{-1} = \sum_{k=1}^{t} \varphi_k \Lambda \varphi_k^T \qquad (2)$$

Divide sum in (2)

$$P_t^{-1} = \sum_{k=1}^{t-1} \varphi_k \Lambda \varphi_k^T + \varphi_t \Lambda \varphi_t^T$$

$$\| \quad P_{t-1}^{-1} = \sum_{k=1}^{t-1} \varphi_k \Lambda \varphi_k^T$$

Hence, recursive formula

$$P_t^{-1} = P_{t-1}^{-1} + \varphi_t \Lambda \varphi_t^T \qquad (4)$$

From (1) we have, divide sum

$$\hat{\theta}_t = P_t \left(\sum_{k=1}^{t-1} \varphi_k \Lambda y_k + \varphi_t \Lambda y_t \right) \qquad (3)$$

we also have from (1)

$$\hat{\theta}_{t-1} = P_{t-1} \sum_{k=1}^{t-1} \varphi_k \Lambda y_k \Rightarrow \sum_{k=1}^{t-1} \varphi_k \Lambda y_k = P_{t-1}^{-1} \hat{\theta}_{t-1}$$

Put in (3)

$$\hat{\theta}_t = P_t \left(P_{t-1}^{-1} \hat{\theta}_{t-1} + \varphi_t \Lambda y_t \right)$$

From (4) $P_{t-1}^{-1} = P_t^{-1} - \varphi_t \Lambda \varphi_t^T$ gives

$$\hat{\theta}_t = P_t \left(\left(P_t^{-1} - \varphi_t \Lambda \varphi_t^T\right) \hat{\theta}_{t-1} + \varphi_t \Lambda y_t \right)$$

$$= \left(I - P_t \varphi_t \Lambda \varphi_t^T\right) \hat{\theta}_{t-1} + P_t \varphi_t \Lambda y_t$$

$$= \hat{\theta}_{t-1} + P_t \varphi_t \Lambda \left(y_t - \varphi_t^T \hat{\theta}_{t-1} \right)$$