Master study
Systems and Control Engineering
Department of Technology
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SCE1106 Control Theory

Solution Exercise 7

Task 1

a) The process model can be written as

$$h_p(s) = \frac{e^{-2s}}{s^2 + 3s + 2} = k\frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)}. \tag{1}$$

where $k = \frac{1}{2}$, $\tau = 2$, $T_1 = 1$ and $T_2 = \frac{1}{2}$. The dominating (largest) time constant in the process is therefore $T_1 = 1$. The integral time constant is then chosen as

$$T_i = T_1 = 1 \tag{2}$$

b) The loop transfer function, $h_0(s)$, is equal to the product of all blocks around the feedback loop (against the signal direction), i.e.,

$$h_0(s) = h_c(s)h_p(s) = K_p\frac{1 + T_i s}{T_i s}k\frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)} = k_0\frac{e^{-\tau s}}{s(1 + T_2 s)} \tag{3}$$

where

$$k_0 = \frac{K_p k}{T_1}, \tag{4}$$

and where we have chosen $T_i = T_1$ in order to simplify the loop transfer function.

The frequency response of the loop transfer function is then obtained by putting $s = j\omega$

$$h_0(j\omega) = k_0\frac{e^{-j\tau\omega}}{j\omega(1 + jT_2\omega)} \tag{5}$$

We write the frequency response on polar form as follows

$$h_0(j\omega) = |h_0(j\omega)|e^{j\angle h_0(j\omega)} \tag{6}$$

where the magnitude is given by

$$|h_0(j\omega)| = \frac{k_0}{\omega\sqrt{1 + (T_2\omega)^2}} \tag{7}$$

and where the phase shift is given by

$$\angle h_0(j\omega) = -\tau\omega - \frac{\pi}{2} - \arctan(T_2\omega) \tag{8}$$

c)

1. The phase crossover frequency, ω_{180}, is given by the frequency, ω, which gives a phase shift equal to $-\pi$, i.e., we have

$$\angle h_0(j\omega) = -\tau\omega - \frac{\pi}{2} - \arctan(T_2\omega) = -\pi \tag{9}$$

This is a non-linear function in the frequency ω. The equation can in this case simply be solved by fiks-point iteration by using the following iteration scheme in a for loop:

$$\omega = \frac{\frac{\pi}{2} - \arctan(T_2\omega)}{\tau} \tag{10}$$

A start value, $\omega = 1$, gives after a few iterations that

$$\omega_{180} = 0.6323 \tag{11}$$

2. We will now chose the proportional constant, K_p, such that the Gain Margin is, $GM = 2$. From the definition of the gain Margin we have that

$$|h_0(j\omega_{180})| = \frac{1}{GM} = \frac{1}{2} \tag{12}$$

Putting in the expression for the magnitude given by Equation (7) we find that

$$\frac{K_p k}{T_1\omega_{180}\sqrt{1+(T_2\omega_{180})^2}} = \frac{1}{2} \tag{13}$$

which gives

$$K_p = \frac{T_1\omega_{180}\sqrt{1+(T_2\omega_{180})^2}}{2k} = 0.6631 \tag{14}$$

3. The gain crossover frequency is given by

$$|h_0(j\omega_c)| = \frac{k_0}{\omega_c\sqrt{1+(T_2\omega_c)^2}} = 1 \tag{15}$$

which can be solved by fiks-point iteration. Se the solution proposal. We find the solution

$$\omega_c = 0.3272 \tag{16}$$

4. The Phase Margin, PM, is then found to be:

$$\begin{aligned} PM &= \angle h_0(j\omega_c) + \pi \\ &= -\tau\omega_c - \frac{\pi}{2} - \arctan(T_2\omega_c) + \pi \\ &= 0.7542 \ [\text{rad}] = 43.21 \ [^\circ] \end{aligned} \tag{17}$$

d) The simulation of the closed loop system with the PI controller settings found can be done as shown in the MATLAB script **losn7_ogg1.m**. As we see, there is more overshot in the output response with this settings compared with the Skogestad settings.

MATLAB-script losn7_oppg1.m

```
% losn7_oppg1.m
% Formaal: Loesning av oppgave 1 i oeving 7 i faget Prosessregulering.
% Inneholder beregning av:
% Fase kryssfrekvens, omega_180.
% Forsterkningsmargin, GM.
% Forsterkningskryssfrekvens, omega_c.
% Fasemargin, PM.
% samt PI-regulator syntese.
% DDiR, 22. oktober 2002

clear all

k=0.5; T1=1; T2=0.5; tau=2;

%% Velger Ti=1;
% Beregning av fasekryssfrekvensen, omega_180,
% ved fikspunktiterasjon.
omega=1;
for i=1:100
   omega=(pi/2-atan(T2*omega))/tau;
end
omega_180=omega                                % Fase kryssfrekvensen.
%Test, vinkel_h0=-pi.
vinkel_h0=-tau*omega-pi/2-atan(T2*omega)

% Beregning av K_p slik at GM=2
Kp=T1*omega_180*sqrt(1+(T2*omega_180)^2)/(2*k)

% Beregning av forsterknings kryssfrekvensen, omega_c,
% vha. fikspunktiterasjon.
omega=1, k0=Kp*k/T1;
for i=1:100
   omega=k0/sqrt(1+(T2*omega)^2);
end
omega_c=omega                                  % Forsterkningskryssfrekvensen.

% Fasemarginen.
vinkel_h0=-tau*omega_c-pi/2-atan(T2*omega_c);
PM=(vinkel_h0+pi)*180/pi



%% sjekk vha control system tbx
Ti=T1;
num1=[0,0,1];
```

```
den1=[1,3,2];
[numd,dend]=pade(tau,10);
[num_hp,den_hp]=series(num1,den1,numd,dend);

num_hc=Kp*[Ti,1];
den_hc=[Ti,0.e-9];

[num_h0,den_h0]=series(num_hc,den_hc,num_hp,den_hp);
[Gm,Pm,W180,Wc] = margin(num_h0,den_h0)
```