Master study
Systems and Control Engineering
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SCE1106 Control Theory

Exercise 5

Task 1

Given a system described by the 2nd order continuous time state space model

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 3 & 4 \end{bmatrix} x \tag{2}$$

Transform the given state space model to the following canonical forms:

1. Controllability canonical form
2. Controller canonical form
3. Observability canonical form
4. Observer canonical form

Specify the transformation matrix, T, in the state transformation $x = Tz$, for each case. A MATLAB script which does the computations should be written.

Task 2

Given a Single Input Single Output (SISO) continuous time process, i.e., a process with one control variable, u, and one process output measurement, y. The process is excited with a step response in the input, u. The process input and output at different discrete time instants are observed and given as in Table 1. Here, k, is discrete time and u_k and y_k is the control variable and the measurement at discrete time k , respectively.

1. Plot the input and output data, i.e., plot u_k and y_k for $k = 0, \dots, 7$ as the x-axis. Use the **plot** function in MATLAB. Store the input and output data in data vectors U and Y respectively.

2. From the known input and output data we want to identify the parameters in the following state space model.

$$x_{k+1} = \phi x_k + \delta u_k \tag{3}$$

$$y_k = x_k \tag{4}$$

   Find the model parameters ϕ and δ. Use the Ordinary Least Squares (OLS) method to find the parameters.

Table 1: Observed data from the process.

k	u_k	y_k
0	0	0
1	1	0
2	1	0.5
3	1	0.7
4	1	0.78
5	1	0.812
6	1	0.8248
7	1	0.8299

3. The input and output data is observed (recorded) with a sampling time $\Delta t = 10$ $[s]$. Find the continuous time constant T of the system and the process gain from u to y.

4. On the basis of the discrete time model you should find the parameters in the corresponding continuous time model

$$\dot{x} = ax + bu \tag{5}$$

$$y = x \tag{6}$$

5. Find the transfer function from u to y, i.e. find the Laplace transformed model.

6. The process is to be controlled so that the output, y, is as close to a reference signal y^s as possible. Write down the expressions for a PI controller both in the Laplace plane (transfer function) and in the state space, i.e. specify the transfer function model and a state space model for a PI controller.

7. How would you chose the integral time constant, T_i? How would you chose the proportional constant K_p? Tips: Chose T_i equal to the time constant in the process. Chose K_p so that the response from the set point to the output, y, have a specified time constant, $T_c = 4$ $[s]$.

8. We have in step 25 above identified a 1st order dynamic model. It is in general low order models which can be properly identified from observed input and output system data. Because the system is sampled in discrete time instants we can argue that we have a time delay equal to at least $\tau = \frac{\Delta t}{2}$, in addition to the observed process dynamics.

   A more realistic process model is therefore to assume that we have a transport delay in addition to the model obtained in step 5 above, hence we have

$$y(s) = \frac{k}{1 + Ts} e^{-\tau s} u(s) \tag{7}$$

where

$$\tau = \frac{\Delta t}{2}. \tag{8}$$

find the PI controller parameters, K_p and T_i, by using the Skogestad method.

9. Write a MATLAB script in order to simulate the system with the PI controller.