Master study
Systems and Control Engineering
Department of Technology
Telemark University College
DDiR, September 8, 2006

SCE1106 Control Theory

Exercise 2

Task 1

Given a process consisting of two serial tanks as shown in the Figure. The two tanks are connected with a valve controlling the flow of liquid q through the two tanks. . Two liquid flows, u_1 and u_2 are added the left and right tanks, respectively. u_1 and u_2 are manipulable (control) variables. The levels x_1 and x_2 in the tanks are the process states. The flow v out of the right tank is handeled as a disturbance in the system. We are assuming that the density ρ $[\frac{kg}{m^3}]$, of the liquid is constant.

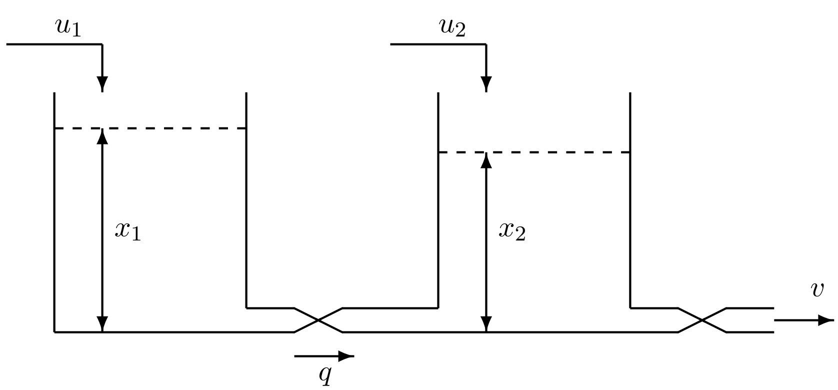

Figure 1: System med to kar i serie.

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x_1	level of the left tank	$[m]$
x_2	level of the right tank	$[m]$
u_1	volume flow of liquid to the left tank	$[\frac{m^3}{s}]$
u_2	volume flow of liquid to the right tank	$[\frac{m^3}{s}]$
q	volume flow of liquid from the left to right tanks	$[\frac{m^3}{s}$
v	volume flow out of the right tank	$[\frac{m^3}{s}]$
A_1	areal of the left tank	$[m^2]$
A_2	areal of the right tank	$[m^2]$

We are assuming that the volume flow through the two tanks, q, are propertional to the level difference of the two tanks and simply modelled by a linear valve characteristic, i.e.,

$$q = k(x_1 - x_2) \tag{1}$$

Note that if one are to obtain a final linear dynamic model for control design, then, a simple linear characteristic at this stage should be chosen.

a) Develop a mathematical model of the process and show that this model can be written in state space form as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix} v \tag{2}$$

or

$$\dot{x} = Ax + Bu + Cv. \tag{3}$$

b) Given the system parameters $k = 1$, $A_1 = 2$, $A_2 = 1$. Find the eigenvalues of the system matrix A. What can be said of the time constants of the system?

c) Compute an eigenvector matrix M such that

$$M^{-1}AM = \Lambda \tag{4}$$

d) Use the transformation $x = Mz$ and show that the following state space model equivalent

$$\dot{z} = \Lambda z + \tilde{B}u \tag{5}$$

is obtained.

What is the expression for $\tilde{B}$?

e) Compute the transition matrix $\Phi(t) = e^{At}$.

f) Assume that $u_1 = u_2 = 0$ and that the levels of the tanks at time zero, i.e. $t_0 = 0$ are $x_1(0) = 1$ and $x_2(0) = 2$. We are also assuming that $v = 0$. Find the solutions $x_1(t)$ og $x_2(t)$. We are in this example locking for the autonomous response of the system, i.e., the response which is driven only of initial values (usually different from zero). Write a MATLAB m-file and simulate the responses of the system states.

g) Assume that $u_1 = u_2 = 0$ and that the levels of the tanks at time zero, i.e. $t_0 = 0$ are $x_1(0) = 1$ and $x_2(0) = 2$. Find the solutions $x_1(t)$ og $x_2(t)$. Assume here that the flow out of the last tank can be modeled by $v = kx_2$. Write a MATLAB m-file and simulate the responses of the system states.

Task 2

a) Write a m-file function and define the pendulum model in Section 1.9, i.e., the model

$$\dot{x}_1 = x_2, \quad (6)$$

$$\dot{x}_2 = -\frac{g}{r}\sin(x_1) - \frac{b}{mr^2}x_2, \quad (7)$$

with $g = 9.81$, $r = 5$, $m = 8$ and $b = 10$.

b) Simulate the model with initial values

$$x(t_0 = 0) = \begin{bmatrix} \frac{\pi}{4} \\ 0 \end{bmatrix} \quad (8)$$

c) How long time will it take before the states is approximately zero?