Task 4 Step responses

a) $h_p(s) = k \dfrac{1}{1+Ts}$, $k = 2$

b) $h_p(s) = k \dfrac{e^{-\tau s}}{s^2}$, $\tau \approx 5$

Double integrator, instable

c) Inverse response + time delay

$$h_p(s) = k \frac{1-T_z s}{(1+T_1 s)(1+T_2 s)} e^{-\tau s}, \quad \tau = 5$$

Inverse response time constant, $T_z > 0$

d) Oscillating plant with gain, $k = 2$, $\tau = 5$

$$h_p(s) = k \frac{e^{-\tau s}}{T_0^2 s^2 + 2\zeta T_0 s + 1} \quad \text{and} \quad 0 < \zeta < 1$$

Visit MATLAB m-file, task_resp2mod17.m

Task 2

a) Model

$$h_p(s) = k\frac{e^{-\tau s}}{(1+T_1 s)(1+T_2 s)}$$

SIMC PID controller on cascade form

$$h_c(s) = \frac{1}{h_p(s)} \frac{\frac{y}{r}}{1-\frac{y}{r}} \quad \text{with} \quad \frac{y}{r} = \frac{e^{-\tau s}}{1+T_c s}$$

and $T_c > 0$ user specified tuning param.

$$h_c = \frac{(1+T_1 s)(1+T_2 s)}{k \cancel{e^{-\tau s}}} \frac{\frac{\cancel{e^{-\tau s}}}{1+T_c s}}{1-\frac{e^{-\tau s}}{1+T_c s}} = \frac{(1+T_1 s)(1+T_2 s)}{k} \frac{1}{1+T_c s - e^{-\tau s}}$$

Approx. $e^{-\tau s} \approx 1-\tau s$ gives

$$h_c = \frac{(1+T_1 s)}{k(T_c+\tau)s}(1+T_2 s) = \frac{T_1}{k(T_c+\tau)} \frac{1+T_1 s}{T_1 s}(1+T_2 s)$$

Similar to PID cascade controller

$$h_c(s) = K_p \frac{1+T_i s}{T_i s}(1+T_d s)$$

with

$$K_p = \frac{T_1}{k(T_c+\tau)} \overset{(T_c=\tau)}{=} \frac{T_1}{2k\tau}, \quad T_i = T_1 \text{ and } T_d = T_2$$

b) The cascade form

$$h_c(s) = k_p \frac{1+T_i s}{T_i s}(1+T_d s)$$

may be written on ideal PID form

$$h_c(s) = k_p' + \frac{k_p'}{T_i' s} + k_p' T_d' s$$

where

$$k_p' = k_p\left(1+\frac{T_d}{T_i}\right), \quad T_i' = T_i\left(1+\frac{T_d}{T_i}\right), \quad T_d' = T_d \frac{1}{1+\frac{T_d}{T_i}}$$

Task 2 (or 3?)

a) SIMC need a model

- $h_p(s) = k\dfrac{e^{-Ts}}{1+T_0 s}$ or $h_p(s) = k\dfrac{e^{-Ts}}{(1+T_1 s)(1+T_2 s)}$ (1)

- Given model
$$h_p(s) = k\frac{e^{-Ts}}{1+Ts} \quad (2)$$

gives PI controller (SIMC) with

$$K_p = \frac{T}{K(T_c+T)} \quad \text{and} \quad T_i = \min(T, 4(T_c+T))$$

or $T_i = T$ if derived from $h_c = \dfrac{1}{h_p}\dfrac{\frac{y}{r}}{1-\frac{y}{r}}$ and model in (2).

b) • $y = h_{ry}(s)\cdot r + h_{vy}(s)\cdot v$

where

- $h_{ry} = \dfrac{h_c h_p}{1+h_c h_p}$, $h_{vy}(s) = \dfrac{h_v}{1+h_c h_p}$

- A desired (ideal) transfer function is
$$h_{ry} = \frac{y}{r} = \frac{e^{-Ts}}{1+T_c s} \quad \text{and} \quad y = h_{ry}\cdot r$$

We have $y \approx r$ at $s \approx 0$ (same as letting time $t \to \infty$)

c)

- $$h_c = \frac{1}{h_p} \frac{\frac{y}{r}}{1-\frac{y}{r}} = \frac{1}{h_p} \frac{\frac{e^{-\tau s}}{1+T_c s}}{1-\frac{e^{-\tau s}}{1+T_c s}}$$

$$= \frac{1}{h_p} \frac{e^{-\tau s}}{1+T_c s - e^{-\tau s}}$$

- SIMC are using approximation

$$e^{-\tau s} \simeq 1 - \tau s$$

d) Given $h_p = k \frac{e^{-\tau s}}{(1+Ts)^4}$ with $\tau = 1$

2nd order model approx. $h_p = k \frac{e^{-\tau s}}{(1+T_1 s)(1+T_2 s)}$

with $T_1 = T$, $T_2 = T + \frac{1}{2}T$, $\tau = 1 + \frac{1}{2}T + T$

Actually, reverse such that $T_1 \geq T_2 \geq 0$ and then

$$T_1 = T + \frac{1}{2}T = \frac{3}{2}T, \quad T_2 = T, \quad \tau = 1 + \frac{1}{2}T + T$$
$$= 1 + \frac{3}{2}T$$

- SIMC PID controller

$$h_c = k_p \frac{1+T_i s}{T_i s}(1+T_d s)$$

$$k_p = \frac{T_1}{k(T_c+\tau)}, \quad T_i = T_1, \quad T_d = T_2 \quad \text{is OK}$$

Task 3 (or 4?)

a) $$h_p = k \frac{e^{-\tau s}}{\tau_0^2 s^2 + 2\zeta \tau_0 s + 1} \qquad (1)$$

- ζ is relative damping, oscillating with $0 \leq \zeta < 1$
- Oscillating with $0 \leq \zeta < 1$
- SIMC PID on ideal form

$$h_c = K_p \left(1 + \frac{1}{T_i s} + T_d s\right)$$

with

$$K_p = \frac{2\tau_0 \zeta}{k(T_c + \tau)}, \quad T_i = 2\tau_0 \zeta, \quad T_d = \frac{\tau_0}{2\zeta}$$

is derived from

$$h_c = \frac{1}{h_p} \frac{\frac{y}{r}}{1 - \frac{y}{r}} \text{ with } (1) \text{ and } \frac{y}{r} = \frac{e^{-\tau s}}{1 + T_c s}.$$

b) Plant model (time delay)

$$h_p = k e^{-\tau s}$$

Controller

$$h_c = \frac{1}{h_p} \frac{\frac{y}{r}}{1-\frac{y}{r}} = \frac{1}{k\cancel{e^{-\tau s}}} \frac{\frac{\cancel{e^{-\tau s}}}{1+T_c s}}{1-\frac{e^{-\tau s}}{1+T_c s}} = \frac{1}{k} \frac{1}{1+T_c s - e^{-\tau s}}$$

$$= \frac{1}{k} \frac{1}{(T_c+\tau)s} = \frac{1}{T_i s}$$

with $T_i = k(T_c+\tau)$

This is an Integral controller.

Simple using an I-controller for pure time delay processes.

c) A plant $h_p = k\frac{e^{-\tau s}}{s}$

gives a pure P-controller with

$$K_p = \frac{1}{k(T_c+\tau)}$$

Proof

$$h_c = \frac{1}{h_p} \frac{\frac{y}{r}}{1-\frac{y}{r}} = \frac{s}{k\cancel{e^{-\tau s}}} \frac{\frac{\cancel{e^{-\tau s}}}{1+T_c s}}{1-\frac{e^{-\tau s}}{1+T_c s}} = \frac{\cancel{s}}{k} \frac{1}{(T_c+\tau)\cancel{s}}$$

d)

$$h_0 = h_c h_p = K_p \frac{1+T_i s}{T_i s} k \frac{e^{-\tau s}}{s} = \frac{K_p k}{T_i} \frac{1+T_i s}{s^2} e^{-\tau s}$$

$$\frac{y}{r} = \frac{h_0}{1+h_0} \quad \text{and} \quad h_0 = \frac{k_p k}{T_i s^2}(1+T_i s) \quad \text{with } e^{-\tau s} \approx 1$$

gives

$$\frac{y}{r} = \frac{\frac{k_p k}{T_i s^2}(1+T_i s)}{1+\frac{k_p k}{T_i s^2}(1+T_i s)} = \frac{k_p k(1+T_i s)}{T_i s^2 + k_p k(1+T_i s)}$$

$$= \frac{1+T_i s}{\frac{T_i}{k_p k}s^2 + T_i s + 1} = \frac{1+T_i s}{\tau_0^2 s^2 + 2\zeta\tau_0 s + 1} = \frac{\rho(s)}{\pi(s)}$$

We have

$$\tau_0^2 = \frac{T_i}{k_p k}, \quad \overbrace{2\zeta\tau_0 = T_i}^{(1)}$$

We have from (1) $4\zeta^2\tau_0^2 = T_i^2$

and

$$4\zeta^2\tau_0^2 = 4\zeta^2\frac{\cancel{T_i}}{k_p k} = T_i^{\cancel{2}}$$

gives

$$T_i = \frac{4\zeta^2}{k_p k} = \frac{4}{k_p k}$$

With $k_p = \dfrac{1}{k(T_c+\tau)}$

$$T_i = \frac{4}{k_p} = 4\cancel{k}(T_c+\tau) = \underline{\underline{4(T_c+\tau)}}$$

Task 4 From Ch. 10.4.2

a)
$$u = k_p e + \frac{k_p}{T_i s} e + k_p T_d s e$$

Defining
$$z = \frac{k_p}{T_i s} e \Rightarrow T_i s z = k_p e$$

and $\dot{z} = \frac{k_p}{T_i} e$

and
$$u = k_p e + z + k_p T_d \dot{e}$$

PID on state space form
$$\boxed{\begin{aligned} \dot{z} &= \frac{k_p}{T_i} e \\ u &= k_p e + z + k_p T_d \dot{e} \end{aligned}}$$

b) Using $\dot{z} \approx \frac{z_{k+1} - z_k}{\Delta t}$ and r - constant,

and then $\dot{e} = \dot{r} - \dot{y} = -\dot{y} \approx - \frac{y_k - y_{k-1}}{\Delta t}$

We have
$$\boxed{\begin{aligned} z_{k+1} &= z_k + \Delta t \frac{k_p}{T_i} e_k \qquad (1) \\ u_k &= k_p e_k + z_k - \frac{k_p T_d}{\Delta t}(y_k - y_{k-1}) \end{aligned}}$$

c) Velocity form $u_k = u_{k-1} + \Delta u_k$

Express

$$\Delta u_k = u_k - u_{k-1} = k_p e_k + z_k - \frac{k_p T_d}{\Delta t}(y_k - y_{k-1})$$
$$- \left(k_p e_{k-1} + z_{k-1} - \frac{k_p T_d}{\Delta t}(y_{k-1} - y_{k-2})\right)$$

$$= k_p e_k + z_k - z_{k-1} - k_p e_{k-1} - \frac{k_p T_d}{\Delta t}(y_k - 2y_{k-1} + y_{k-2})$$

Use

$$z_k - z_{k-1} = \Delta t \frac{k_p}{T_i} e_{k-1} \quad \text{from (1)}$$

gives answer

$$\Delta u_k = k_p e_k + \left(\Delta t \frac{k_p}{T_i} - k_p\right) e_{k-1} - \frac{k_p T_d}{\Delta t}(y_k - 2y_{k-1} + y_{k-2})$$

We may write

$$\Delta u_k = g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2})$$

when

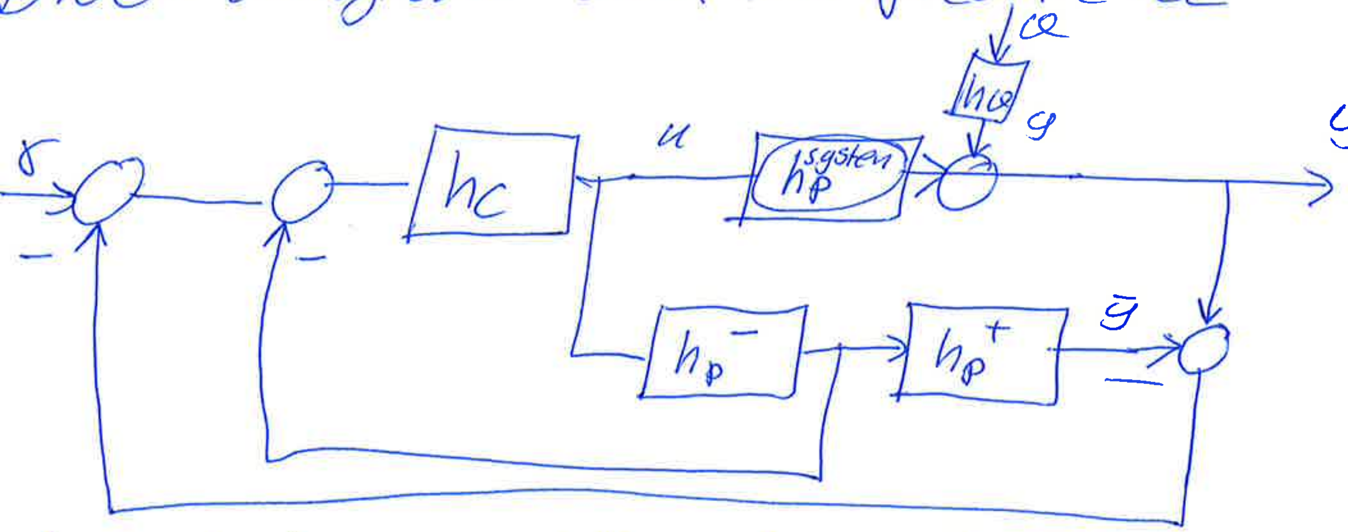
$$g_0 = k_p, \quad g_1 = \Delta t \frac{k_p}{T_i} - k_p = k_p\left(\frac{\Delta t}{T_i} - 1\right)$$
$$g_2 = -\frac{k_p T_d}{\Delta t}$$

Δt – sampling interval

d) No, we do not need model inorder to use a PID controller.

Task 5 Smith predictor Ch. 13 11/11

Block diagram Smith predictor



a) • Systems with large time delay
• Block diagram as above
• Split model

$$h_p = h_p^- \cdot h_p^+$$

when h_p^- part of model without time delay $e^{-\tau s}$, invers response $1-\tau s$ etc

h_p^+ the irrational part, $e^{-\tau s}$ etc.

b) $$h_r = \frac{h_c h_p^s}{1 + h_p^- h_c + (h_p^s - h_p) h_c}$$

c) $$h_d = \frac{1 + h_c(h_p^- - h_p) h_v}{1 + h_p^- h_c + (h_p^s - h_p) h_c}$$

Here

$$y = h_r r + h_d v$$

Visit lecture notes p. 153 and 152