Task 1 (20%): System dynamics: From response to model

Consider a process described by the transfer function model

$$y = h_p(s)u. \tag{1}$$

The transfer function $h_p(s)$ could be a function of the system gain K, time delay τ, system transmission zero τ_z and time constants T_1 and/or T_2.

a) Given the input and output transient responses, $u = u(t)$ and $y = y(t)$ respectively, as illustrated in Figure 1. Propose the structure of the corresponding transfer function $h_p(s)$?

b) Given the input and output transient responses, $u = u(t)$ and $y = y(t)$ respectively, as illustrated in Figure 2. Propose the structure of the corresponding transfer function $h_p(s)$?

c) Given the input and output transient responses, $u = u(t)$ and $y = y(t)$ respectively, as illustrated in Figure 3. Propose the structure of the corresponding transfer function $h_p(s)$?

d) Given the input and output transient responses, $u = u(t)$ and $y = y(t)$ respectively, as illustrated in Figure 4. Propose the structure of the corresponding transfer function $h_p(s)$?

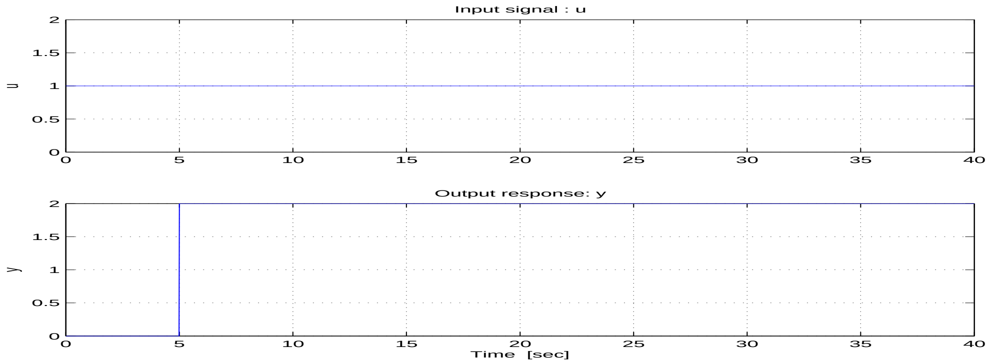

Figure 1: Time response of dynamic system from input u to output y.

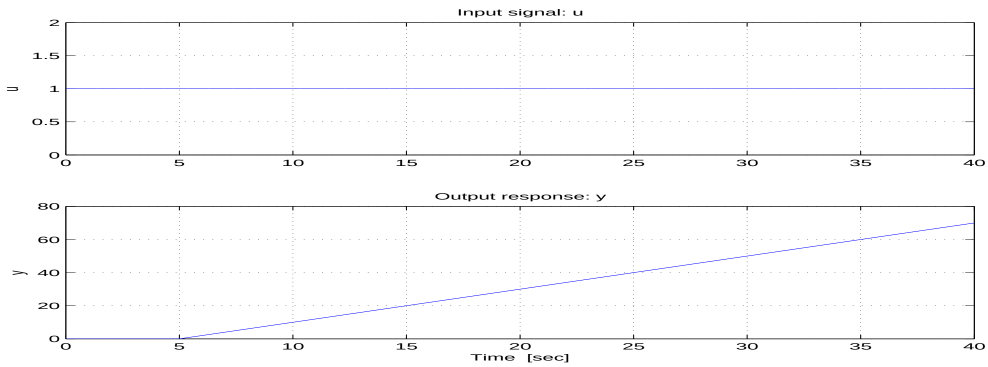

Figure 2: Time response of dynamic system from input u to output y.

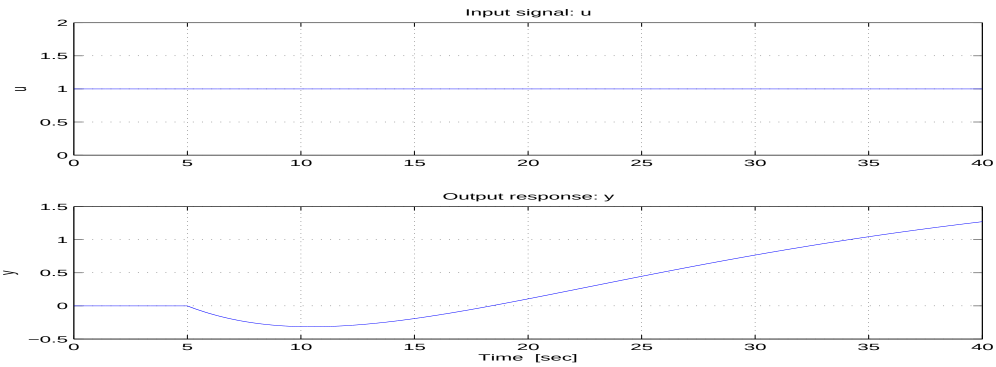

Figure 3: Time response of dynamic system from input u to output y.

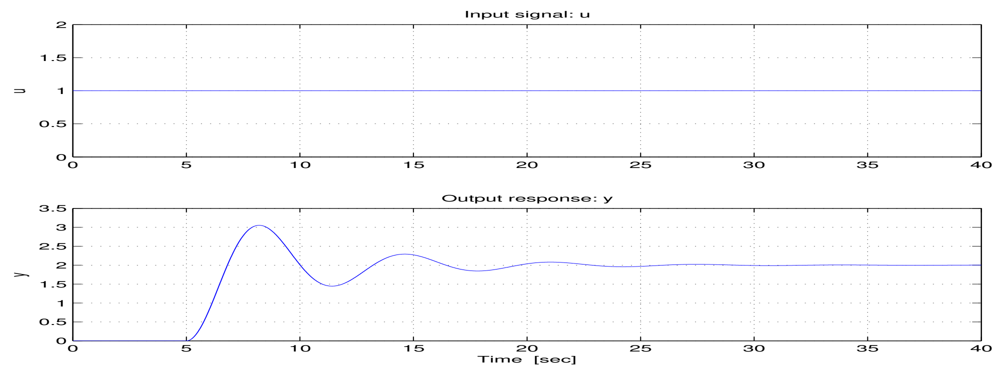

Figure 4: Time response of dynamic system from input u to output y.

Task 2 (20%): PID-control, the SIMC method

Consider a process described by the transfer function model

$$y = h_p(s)u + h_v(s)c. \tag{2}$$

The process is controlled by a controller of the form

$$u = h_c(s)(r - y). \tag{3}$$

The feedback control system is illustrated in Figure (5).

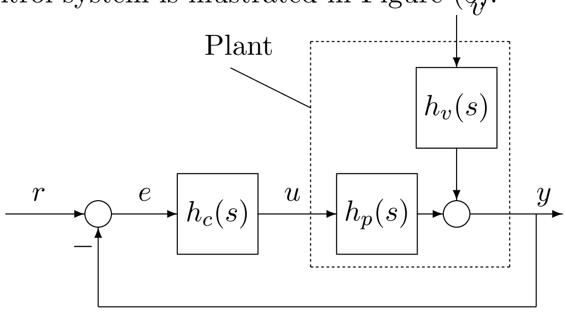

Figure 5: Standard feedback system. Plant described by transfer functions $h_p(s)$ and $h_v(s)$, controller with transfer function $h_c(s)$.

a) Give a short description of the SIMC (also known as the Skogestad) method for tuning PID controllers. Comment in particular upon:

- Model requirement.
- As an illustrative example use SIMC tuning on a model $h_p(s) = K\frac{e^{-\tau s}}{1+Ts}$.

b) Consider the feedback control system in Figure (5).

- Find the transfer functions $h_{ry}(s)$ and $h_{vy}(s)$ from the reference, r, and the disturbance v to the output measurement, y, in the model

$$y = h_{ry}(s)r + h_{vy}(s)v. \tag{4}$$

  I.e. find expressions for $h_{ry}(s)$ and $h_{vy}(s)$?
- Assume $v = 0$. Specify a desired transfer function $h_{ry}(s)$ for the controlled system (closed loop set-point response)? We want $y \approx r$.

c)

- Find an expression for the controller transfer function, $h_c(s)$, as a function of the time delay transfer function $e^{-\tau s}$ and the model transfer function $h_p(s)$ for the process.
- Comment upon the approximation for $e^{-\tau s}$ used in SIMC ? Find the resulting controller transfer function $h_c(s)$?

d) Given a 4th order process model as follows

$$h_p(s) = K\frac{e^{-s}}{(1+Ts)^4}. \quad (5)$$

with gain K and time constant T with multiplicity $n = 4$.

- Use the half rule for model reduction and formulate a 2nd order model approximation of the form

$$h_p(s) = K\frac{e^{-\tau s}}{(1+T_1 s)(1+T_2 s)}, \quad (6)$$

for the transfer function in Equation (5). Find the gain K, the time delay τ and the time constants T_1 and T_2 in the model approximation Equation (6) ?
- Find the controller $h_c(s)$ by the SIMC (Skogestad) method. What type of controller is this?

Task 3 (25%): PID, SIMC, Frequency analysis

a) Assume that the process, $h_p(s)$, is modeled by a 2nd order process of the form

$$h_p(s) = k\frac{e^{-\tau s}}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \quad (7)$$

- What are the definitions for the parameters ξ and τ_0 in in the model (7).
- When is the process oscillating ?
- Find the controller $h_c(s)$ by the SIMC (Skogestad) method.
- What type of controller is this?

b) Given a plant $y = h_p(s)u$ with model

$$h_p(s) = ke^{-\tau s}. \quad (8)$$

Find a reasonable controller $h_c(s)$ for this plant described by (8) ?

c) Given a plant $y = h_p(s)u$ with model as a pure time delay

$$h_p(s) = k\frac{e^{-\tau s}}{s}. \quad (9)$$

Find a P-controller $h_c(s)$ for this plant described by (9) ?

d) Given a feedback system as illustrated in Figure (5).

Consider a PI controller, $h_c(s)$, and an integrating plus time delay process, $h_p(s)$, given by

$$h_c(s) = K_p\frac{1+T_i s}{T_i s}, \quad h_p(s) = k\frac{e^{-\tau s}}{s}, \quad (10)$$

where K_p and T_i are the PI controller parameters, k is the gain velocity (slope of the integrator) and τ the time delay.

- Write down an expression for the loop transfer function, $h_0(s)$.

e) Assume for simplicity that we use an approximation $e^{-\tau s} \approx 1$ for the time delay (same as neglecting the time delay in the model, Eq. (10)).

The set-point response transfer function may then be written as

$$\frac{y}{r} = \frac{\rho(s)}{\pi(s)} \tag{11}$$

where the characteristic polynomial $\pi(s)$ may be written on standard 2nd order form as follows

$$\pi(s) = \tau_0^2 s^2 + 2\zeta\tau_0 s + 1. \tag{12}$$

Here τ_0 is the response time and ζ the relative damping coefficient.

- Find expressions for the coefficients τ_0^2 and $2\zeta\tau_0$ in the characteristic polynomial Eq. (12) as a function of the PI controller parameters K_p, T_i and the gain velocity parameter k.
- Assume that we prescribe a unit relative damping, i.e. $\zeta = 1$. Find expressions for the PI controller parameters K_p and T_i as a function of a prescribed response time $\tau_0 > 0$. Notice: Use the P-controller from 3c) as K_p !

Task 4 (20%): PID controller

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s = K_p + \frac{K_p}{T_i s} + K_p T_d s, \tag{13}$$

such that the control is generated by

$$u(s) = h_c(s) e(s) \tag{14}$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, r, in this task.

a) Write down a continuous state space model for the PID controller in Equations (13) and (14).

b) Find a discrete time state space model for the PID controller in Step 4a) above.

Use the explicit Euler method for discretization.

c) In this subtask you should use the trapezoid method for discretization.

Find a discrete time PID controller in Step 4b) above on so called velocity (incremental, deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}). \tag{15}$$

You should also write down the expressions for the parameters g_0, g_1 and g_2.

d) Do you need a model in order to use a PID controller ? The answer is: Yes or No

Task 5 (15%): Smith predictor

Assume in this task a system described by the following transfer function model

$$y = h_p(s)u + h_v(s)v. \tag{16}$$

a) Answer the following:

- When may it make sense to use a Smith predictor?
- Sketch a block diagram of a system controlled by a Smith predictor.
- Give a short description of the different elements in the Smith predictor.

b) Find the transfer function from the reference r to the system output y in the Smith predictor.

c) Find the transfer function from the disturbance v to the system output y in the Smith predictor.