# Final Exam
# Solution proposal final exam
# IA1117 Control theory with implementation
# (theory part)
# Tuesday December 12, 2017

January 15, 2018

## Task 1 (20%): System dynamics: From response to model

**a)** A pure time-delay $y = h_p(s)u$ with

$$h_p(s) = Ke^{-\tau s}, \tag{1}$$

with $K = 2$ and $\tau = 5$.

**b)** An integrating plus time-delay plant $y = h_p(s)u$ with

$$h_p(s) = k\frac{e^{-\tau s}}{s}, \tag{2}$$

with velocity gain approx $k = 2$ and $\tau = 5$.

**c)** A two time constant system with inverse response $y = h_p(s)u$ with

$$h_p(s) = K\frac{1 + \tau_z s}{(1 + T_1 s)(1 + T_2 s)}e^{-\tau s}, \tag{3}$$

.

**d)** A time delay oscillating system $y = h_p(s)u$ with

$$h_p(s) = K\frac{e^{-\tau s}}{\tau_0^2 s^2 + 2\xi\tau_0 s + 1}, \tag{4}$$

with relative damping $\xi = 0.2$. Should have relative damping $0 < \xi < 1$.

# Task 2 (20%): PID control, the SIMC method

**a)** The SIMC method as in syllabus, the Skogestad (2003) paper and Ch. 3 lecture notes.

SIMC usually for time constant models and using the half rule for reducing to a 1st order or a 2nd order plus time delay model.

The controller is calculated from

$$h_c(s) = \frac{1}{h_p(s)} \frac{\frac{y}{r}}{1 - \frac{y}{r}} \tag{5}$$

where the closed loop setpoint response from $r$ to $y$ is specified as (the best we may achieve with fedback control if we want $y$=$r$)

$$\frac{y}{r} = \frac{e^{-\tau s}}{1 + T_c s}, \tag{6}$$

where $T_c$ is the tuning parameter, i.e. a user specified (approximate) time constant for the closed loop system. Rule of thumb chosen as $T_c \geq \tau$ and the simple choice $T_c = \tau$ is a robust lower bound.

In case of a 1st order time constant plus time delay model SIMC gives a PI controller with settings

$$K_p = \frac{T}{K(T_c + \tau)}, \;\; T_i = \min(T, 4(T_c + \tau)) \;\; T_c \geq \tau \tag{7}$$

In case that a PID controller is wanted tje plant model should be reduced to a 2nd order time constant plus time delay model

$$h_p(s) = K \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)} \;\; T_1 \geq T_1 \tag{8}$$

or a double integrator plus time delay model. The resulting SIMC controller is then a PID controller on series/cascade form, i.e.,

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}(1 + T_d s) \tag{9}$$

with settings as in Eqs. (7) and with $T_d = T_2$. The PID series/cascade form Eq. (9) may be converted to conventional ideal PID controller form.

**b)**

We have

$$h_{vy}(s) = \frac{h_v}{1 + h_c h_p}, \tag{10}$$

$$h_{ry}(s) = \frac{h_c h_p}{1 + h_c h_p}. \tag{11}$$

**c)** We obtain, by using the specified settpoint response Eq. (6)

$$h_c(s) = \frac{1}{h_p(s)} \frac{\frac{y}{r}}{1 - \frac{y}{r}} = \frac{1}{h_p(s)} \frac{e^{-\tau s}}{1 + T_c s - e^{-\tau s}}. \tag{12}$$

In SIMC the simple series approximation to the matrix exponential is used, i.e.

$$e^{-\tau s} \approx 1 - \tau s. \tag{13}$$

This gives

$$h_c(s) = \frac{1}{h_p(s)} \frac{1 - \tau s}{(T_c + \tau) s}. \tag{14}$$

Further expressions depends on the plant model $h_p(s)$.

**d)** The half rule gives a 2nd order plus time delay model

$$h_p(s) = K \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)}, \tag{15}$$

where $T_1 = T$, $T_2 = T + 0.5T = \frac{3}{2}T$, $\tau := \tau + 0.5T + T = \tau + \frac{3}{2}T$. SIMC gives a series/cascade PID controller as discussed in 1a).

## Task 3 (25%): PID, SIMC frequency analysis

**a)** Lecture notes Ch. 3.5

**b)** A pure I-controller

$$h_c(s) = \frac{1}{T_i s}, \tag{16}$$

with

$$Ti = k(T_c + \tau), \tag{17}$$

where $T_c \geq \tau$ is a the tuning parameter (approximate time constant of the closed loop system).

**c)** We find a P-controller $h_c(s) = K_p$ with

$$K_p = \frac{1}{k(T_c + \tau)}, \tag{18}$$

where $T_c \geq \tau$ is a the tuning parameter.

**d)** The loop transfer function

$$h_0(s) = h_c(s)h_p(s) = \frac{kK_p}{T_i}\frac{1 + T_i s}{s^2}e^{-\tau s}. \tag{19}$$

**e)** Theory in Ch. 3.8 and 3.8.1 We find

$$\tau_0^2 = \frac{T_i}{K_p k}, \quad 2\xi\tau_0 = T_i. \tag{20}$$

This gives

$$K_p = \frac{T_i}{\tau_0^2 k} = \frac{2\xi}{\tau_0 k}, \quad T_i = 2\xi\tau_0. \tag{21}$$

Using the P-controller from 3c) gives

$$T_i = 4\xi^2(T_c + \tau). \tag{22}$$

## Task 4 (20%): PID controller

**a)** Lecture notes Ch. 4.2.2

**b)** Lecture notes Ch. 4.2.3

**c)** Lecture notes Ch. 10.4.3

**d)** Lecture notes Ch. 10.4.3 We find the the controller formulation

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}) \tag{23}$$

where

$$g_0 = K_p(1 + \frac{\Delta t}{2T_i}),\ \ g_1 = -K_p(1 - \frac{\Delta t}{2T_i}),\ \ g_2 = -\frac{K_p T_d}{\Delta t}. \tag{24}$$

**e)** No. We do not need a model in order to use a PID controller.

## Task 5 (20%): The Smith predictor

**a)** Lecture notes Ch. 13.2 and Figure 13.1

**b)** Lecture notes Ch. 13.2.1 and Eq. (13.4).

$$h_r(s) = \frac{h_c h_p}{1 + h_m^- h_c + (h_p - h_m)h_c}. \tag{25}$$

**b)** Lecture notes Ch. 13.2.1 and Eq. (13.3).

$$h_d(s) = \frac{(1 + h_c(h_m^- - h_m))h_v}{1 + h_m^- h_c + (h_p - h_m)h_c}, \tag{26}$$