Final Exam
Course SCE1106 Control theory with implementation (theory part)
Monday December 12, 2011
kl. 9.00-12.00

December 6, 2011

Task 1 (8%): PID-control, the SIMC method

Consider a process described by the transfer function model

$$y = h_p(s)u. \tag{1}$$

The process are to be controlled by a controller of the form

$$u = h_c(s)(r - y). \tag{2}$$

The feedback control system is illustrated in Figure (1).

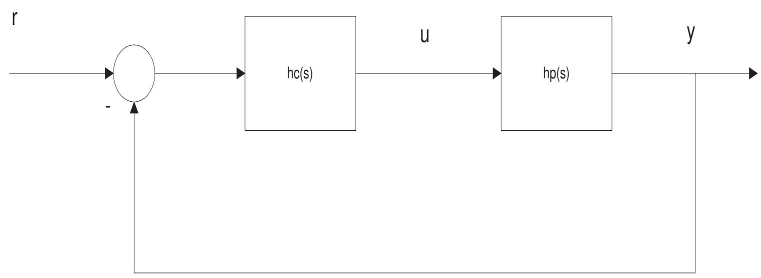

Figure 1: Standard feedback control system.

a) Consider the feedback control system in Figure (1).

- Find the transfer function from the reference, r, to the output measurement, y, i.e., find the transfer function

$$\frac{y}{r} = h_{ry}(s) \tag{3}$$

  where $h_{ry}(s)$ is the transfer function from r to y.
- Find an expression for the transfer function, $h_c(s)$, for the controller as a function of the ratio $\frac{y}{r}$ and the transfer function for the process, $h_p(s)$.

We will in the following subtasks specify that the set point response from the reference, r, to the output, y, should be given by

$$\frac{y}{r} = \frac{e^{-\tau s}}{1 + T_c s} \tag{4}$$

where $T_c \geq \tau$ is a user specified time constant and $\tau > 0$ is the time delay. You may use the simple approximation $e^{\tau s} \approx 1 - \tau s$ in the algebraic calculations when needed.

b) Assume that the process, $h_p(s)$, is modeled by a 2nd order transfer function given by

$$h_p(s) = k\frac{e^{-\tau s}}{(1+T_1 s)(1+T_2 s)}, \tag{5}$$

where $T_1 > T_2 > 0$.

Find the controller $h_c(s)$ by the SIMC (Skogestad) method. What type of controller is this?

c) Assume that the process, $h_p(s)$, is modelled by a 2nd order oscillating process of the form

$$h_p(s) = k\frac{e^{-\tau s}}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \tag{6}$$

Find the controller $h_c(s)$ by the SIMC (Skogestad) method. What type of controller is this?

d) Given a process with a model as follows

$$y = h_p(s)u, \tag{7}$$

where

$$h_p(s) = k\frac{e^{-\tau s}}{(1+Ts)^3}. \tag{8}$$

and $T > 0$.

Use the half rule for model reduction and formulate a 1st order model approximation of the form

$$h_p(s) = k\frac{e^{-\tau s}}{1+Ts}, \tag{9}$$

for the transfer function in Equation (8).

Task 2 (8%):
PI control of pure integrating process

Consider the feedback system in Figure 1 and that the process have a small time delay and that we approximate the process with a pure integrating process with transfer function model

$$h_p(s) = \frac{k}{s}, \tag{10}$$

where k is the gain velocity (slope of the integrator).
We want to control the pure integrating process with a PI controller in order to obtain integral action for load disturbances at the input.

a) Use the formula for the controller, $h_c(s)$, found in task 1a) and find an expression for the proportional gain, K_p, in a P-controller in terms of a prescribed time constant, T_c, for the set-point response.

b) Let us use the proportional gain, K_p, in the P-controller found in step 2a) above and let us find the integral time constant, T_i, such that the pole polynomial

$$1 + h_c(s)h_p(s), \tag{11}$$

of the set-point response $\frac{y}{r} = \frac{h_c h_p}{1+h_c h_p}$ is written on standard form as

$$\pi(s) = \tau_0^2 s^2 + 2\xi\tau_0 s + 1, \tag{12}$$

where τ_0 is the response time and ζ the relative damping coefficient.

Tips: Find expressions for τ_0^2 and $2\xi\tau_0$ and solve for the integral time constant T_i.

Task 3 (25%): Frequency analysis

Given a feedback system as illustrated in Figure 2.

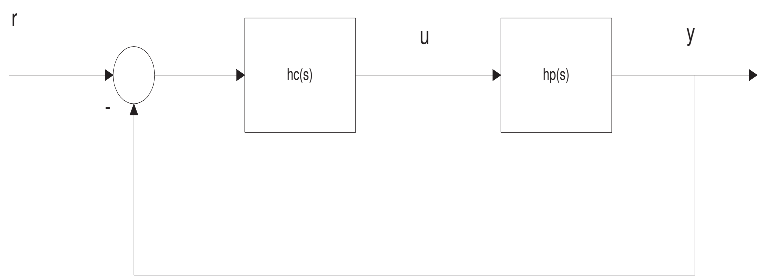

Figure 2: Standard feedback control system.

a) Consider an PI controller, $h_c(s)$, and an integrating plus time delay process, $h_p(s)$, given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}, \quad h_p(s) = k\frac{e^{-\tau s}}{s}, \tag{13}$$

where K_p and T_i is the PI controller parameters, k is the gain velocity (slope of the integrator) and τ the time delay.

- Write down an expression for the loop transfer function, $h_0(s)$.

b) Assume in this subtask 3b) that we use an approximation $e^{-\tau s} \approx 1$ for the time delay (same as neglecting the time delay in the model, Eq. (13).

The set-point response transfer function may then be written as

$$\frac{y}{r} = \frac{h_0}{\pi(s)} \tag{14}$$

where the characteristic polynomial $\pi(s)$ may be written on standard 2nd order form as follows

$$\pi(s) = \tau_0^2 s^2 + 2\zeta\tau_0 s + 1. \tag{15}$$

Here τ_0 is the response time and ζ the relative damping coefficient.

- Find expressions for the coefficients τ_0^2 and $2\zeta\tau_0$ in the characteristic polynomial Eq. (15) as a function of the PI controller parameters K_p, T_i and the gain velocity parameter k.
- Assume that we prescribe a unit relative damping, i.e. $\zeta = 1$. Find expressions for the PI controller parameters K_p and T_i as a function of a prescribed response time $\tau_0 > 0$.

c)

- Find an expression for the frequency response of the loop transfer function $h_0(s)$ found in subtask 3a), on polar form, i.e., such that

$$h_0(j\omega) = |h_0(j\omega)|e^{j\angle h_0(j\omega)}. \tag{16}$$

  Find expressions for the magnitude $|h_0(j\omega)|$ and the phase angle $\angle h_0(j\omega)$.
  Note: You should in this subtask 3c) use the integrator plus time delay model as in Eq. (13).
- Describe the Bode stability criterion !

d)

- Find an expression for the phase crossover frequency, ω_{180}, for the closed loop system ?
- Find an expression for the Gain Margin (GM) ?
- What is the interpretation of the GM ?

e)

- What is the definition of, and how can the gain crossover frequency, ω_c, be computed?
- Find an expression for the Phase Margin (PM) ?
- What is the interpretation of the PM ?

Task 4 (15%): PID controller

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s = K_p + \frac{K_p}{T_i s} + K_p T_d s, \tag{17}$$

such that the control is generated by

$$u(s) = h_c(s) e(s) \tag{18}$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, r, in this task.

a) Write down a continuous state space model for the PID controller in Equations (17) and (18).

b) Find a discrete time state space model for the PID controller in Step 4a) above. Use the explicit Euler method for discretization.

c) Write the discrete time PID controller in Step 4d) above on so called velocity (incremental, deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}). \tag{19}$$

You should also write down the expressions for the parameters g_0, g_1 and g_2.

Task 5 (4%): Smith predictor

Given a system described by the following transfer function model

$$y = h_p(s) u + h_v(s) v \tag{20}$$

a) Answer the following:

- When may it make sense to use a Smith predictor?
- Sketch a block diagram of a system controlled by a Smith predictor.
- Give a short description of the different elements in the Smith predictor.