Final Exam
Course SCE1106 Control theory with implementation (theory part)
Monday December 7th 2009
kl. 9.00-12.00

December 3, 2009

Task 1 (12%): PID-control, the Skogestad method

Consider a process described by the transfer function model

$$y = h_p(s)u. \tag{1}$$

The process are to be controlled by a controller of the form

$$u = h_c(s)(r - y). \tag{2}$$

The feedback control system is illustrated in Figure (1).

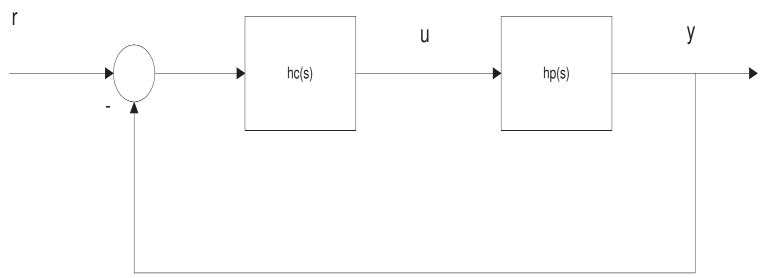

Figure 1: Standard feedback control system.

a) Consider the feedback control system in Figure (1).

- Find the transfer function from the reference, r, to the output measurement, y, i.e., find the transfer function

$$\frac{y}{r} = h_r(s) \tag{3}$$

  where $h_r(s)$ is the transfer function from r to y.

- Find an expression for the transfer function, $h_c(s)$, for the controller as a function of the ratio $\frac{y}{r}$ and the transfer function for the process, $h_p(s)$.

We will in the following subtasks specify that the set point response from the reference, r, to the output, y, should be given by

$$\frac{y}{r} = \frac{1 - \tau s}{1 + T_c s} \tag{4}$$

where T_c is a user specified time constant.

b) Assume that the process, $h_p(s)$, is modelled by a 2nd order transfer function given by

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)}, \tag{5}$$

where $T_1 > T_2 > 0$.

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

c) Assume that the process, $h_p(s)$, is modelled by a 1st order model given by

$$h_p(s) = k\frac{1-\tau s}{1+T_1 s}. \tag{6}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

d) Assume that the process, $h_p(s)$, is modelled by a 2nd order oscillating process of the form

$$h_p(s) = k\frac{1-\tau s}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \tag{7}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

e) Assume that the process is modelled by a pure time delay, i.e. with a process model

$$h_p(s) = ke^{-\tau s}. \tag{8}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

f) Given a process with a model as follows

$$y = h_p(s)u, \tag{9}$$

where

$$h_p(s) = k\frac{e^{-\tau_0 s}}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}. \tag{10}$$

and $T_1 > T_2 > T_3 > 0$.

Use the half rule for model reduction and formulate a 1st order model approximation of the form

$$h_p(s) = k\frac{e^{-\tau s}}{1+Ts}, \tag{11}$$

for the transfer function in Equation (10).

Task 2 (10%): Frequency analysis

Given a feedback system as illustrated in Figure 2.

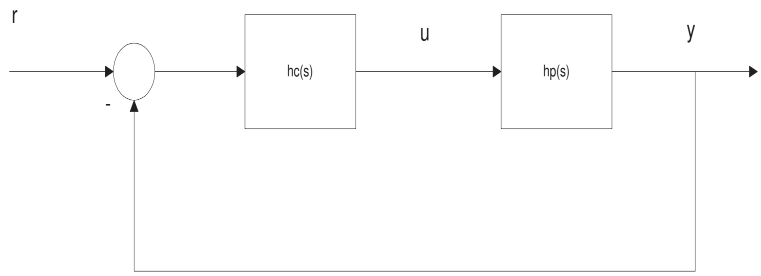

Figure 2: Standard feedback control system.

a) Consider an PID controller on cascade form, $h_c(s)$, and a process, $h_p(s)$, given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}(1 + T_d s), \quad h_p(s) = k \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)}, \tag{12}$$

where $T_1 > T_2 > 0$.

- Write down an expression for the loop transfer function, $h_0(s)$.
- Chose the integral time T_i, and the derivative time, T_d, such that the loop transfer function can be written as follows

$$h_0(s) = k_0 \frac{e^{-\tau s}}{s}. \tag{13}$$

- Write also down the expression for k_0.

b)

- Write the frequency response of the loop transfer function in (13) on polar form, i.e., such that

$$h_0(j\omega) = |h_0(j\omega)| e^{j\angle h_0(j\omega)}. \tag{14}$$

- Describe the Bode stability criterion !

c) Find the phase crossover frequency, ω_{180}, for the system.

d) Find a proportional gain, K_p, such that the closed loop system have a Gain Margin, $GM = \frac{3}{2}$.

e)

- What is the definition of, and how can the gain crossover frequency, ω_c, be computed?
- What is the gain crossover frequency, ω_c, for the above feedback control system?
- What is the corresponding Phase Margin (PM) ?

Task 3 (10%): PID controller

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s = K_p + \frac{K_p}{T_i s} + K_p T_d s, \tag{15}$$

such that the control is generated by

$$u(s) = h_c(s) e(s) \tag{16}$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, r, in this task.

a) Write down a continuous state space model for the PID controller in Equations (15) and (16).

b) Given a linear system

$$\dot{x} = Ax + Bu \tag{17}$$

where $x(t_0)$ is a specified initial state vector.

Write down an expression for the solution, $x(t)$?

c) Use the result from task 3b) above in order to find an expression for the control signal, $u(t)$. Tips: Here you should find a formulation of, $u(t)$, in terms of among others an integral expression.

d) Find a discrete time state space model for the PID controller in Step 2a) above. Use the explicit Euler method for discretization.

e) Write the discrete time PID controller in Step 2d) above on so called incremental (deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}). \tag{18}$$

You should also write down the expressions for the parameters g_0, g_1 and g_2.

Task 4 (3%): Smith predictor and RGA analysis

Given a system described by the following transfer function model

$$y = h_u(s)u + h_v(s)v \tag{19}$$

a) Answer the following:

- When may it make sense to use a Smith predictor?
- Sketch a block diagram of a system controlled by a Smith predictor.

b) Consider a MIMO system, $y = H_p(s)u$. Describe how we can use RGA analysis in order to solve the input and output pairing problem for an 2×2 system, i.e. a system with $r = 2$ control variables in, u, and $m = 2$ outputs in, y. Tips: The answer should include a formula for the RGA matrix, Λ, an a table with rules for the pairing strategy.