

**Sluttprøve**  
**Fag A3494 Prosessregulering**  
**Thursday January 9th 2007**  
**kl. 9.00-12.00**

Sluttprøven består av: 3 oppgaver.  
Oppgaven teller 70 % av sluttkarakteren.

Det er 3 sider i delprøven.

Tillatte hjelpemidler: ark og skrivesaker

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## Task 1 (20%): Frequency analysis

Given a feedback system as illustrated in Figure 1.

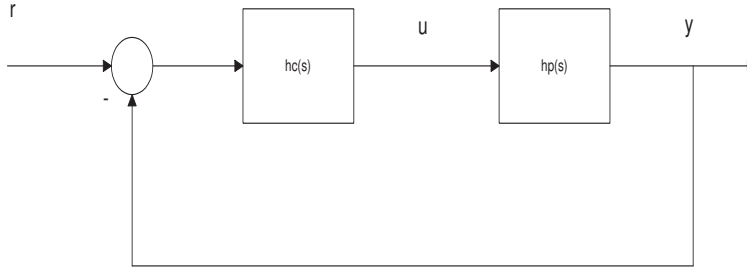


Figure 1: Standard feedback control system.

- Write down an expression for the loop transfer function,  $h_0(s)$ .
- Consider an PID controller on cascade form,  $h_c(s)$ , and a process,  $h_p(s)$ , given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} (1 + T_d s), \quad h_p(s) = k \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)}, \quad (1)$$

where  $T_1 > T_2 > 0$ . Chose the integral time  $T_i$ , and the derivative time,  $T_d$ , such that the loop transfer function can be written as follows

$$h_0(s) = k_0 \frac{e^{-\tau s}}{s}. \quad (2)$$

Write also down the expression for  $k_0$ .

- Write the frequency response of the loop transfer function in (2) on polar form, i.e., such that
$$h_0(j\omega) = |h_0(j\omega)| e^{j\angle h_0(j\omega)}. \quad (3)$$
  - What is the size  $|h_0(j\omega)|$  denoted?
  - What is the size  $\angle h_0(j\omega)$  denoted?
- Find the phase crossover frequency,  $\omega_{180}$ , for the system.
- Find a proportional gain,  $K_p$ , such that the closed loop system have a Gain Margin,  $GM = 2$ .

f)

- What is the definition of, and how can the gain crossover frequency,  $\omega_c$ , be computed?
- What is the gain crossover frequency,  $\omega_c$ , for the above feedback control system?

g)

- What is the definition of, and how can the phase margin,  $PM$ , be computed?
- What is the phase margin,  $PM$ , for the above feedback control system?

h) Consider now a PI controller,  $h_c(s)$ , and a process,  $h_p(s)$ , given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}, \quad h_p(s) = k \frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)(1 + T_3 s)}. \quad (4)$$

where  $T_1 > T_2 > T_3$ .

- Chose the integral time,  $T_i$ , such that the loop transfer function,  $h_0(s)$ , can be written as follows

$$h_0(s) = k_0 \frac{1 - \tau s}{s(1 + T_2 s)(1 + T_3 s)}. \quad (5)$$

Write also down the expression for  $k_0$ .

- Write down the frequency response on polar form of the loop transfer function in Equation (5).

## Task 2 (10%): PID regulator

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s, \quad (6)$$

such that the control is generated by

$$u(s) = h_c(s)e(s) \quad (7)$$

where  $e(s) = r - y(s)$  is the control error. We are assuming a constant reference signal,  $r$ , in this task.

- a) Write down a continuous state space model for the PID controller in Equations (6) and (7).
- b) Find a discrete time state space model for the PID controller in Step 2a) above. Use the explicit Euler method for discretization.
- c) Write the discrete time PID controller in Step 2b) above on so called incremental (deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}). \quad (8)$$

You should also write down the expressions for the parameters  $g_0$ ,  $g_1$  and  $g_2$ .

- d) Show that the solution of the continuous state space model for the PID controller in Step 2a) above (over the time interval from  $t_0$  to  $t$ ) can be written as

$$u(t) = K_p e + z(t_0) + \frac{K_p}{T_i} \int_{t_0}^t e(\tau) d\tau + K_p T_d \dot{e} \quad (9)$$

where  $z(t_0)$  is the initial state for the state equation in the PID controller found in Step 2a).

### Task 3 (5%): Various questions

- a) What is a Smith predictor? When is it suitable to use a Smith predictor? Sketch a block diagram of a system controlled by a Smith predictor.
- b) What is meant by RGA analysis? Write down a formula for computing the RGA matrix. What can the RGA matrix be used for?