Partial test
SCE1106 Control Theory
Friday 20. October 2006
kl. 8.15-10.15, Rom A189

The test consists of 4 tasks.
The test counts $15 = 0.5 * 30$ % of the final grade
in SCE1106 Control with implementation.
The test consists of three pages.
Aid: paper and pen.

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Task 1 (14%): PID-control, the Skogestad method

We are going to study a process described by the transfer function model

$$y = h_p(s)u. \tag{1}$$

The process are to be controlled by a controller of the form

$$u = h_c(s)(r - y). \tag{2}$$

The feedback control system is illustrated in Figure (1).

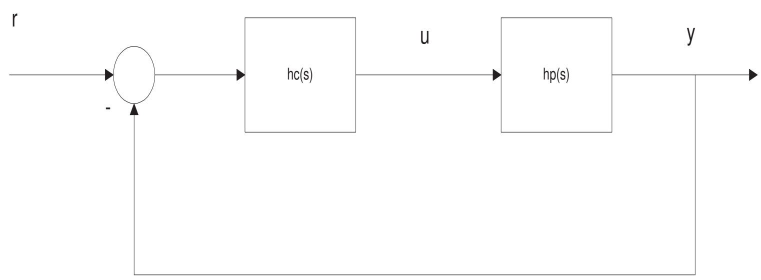

Figure 1: Standard feedback control system.

a) Consider the feedback control system in Figure (1). The transfer function from the reference, r, to the output measurement, y, is given by

$$\frac{y}{r} = \frac{h_p h_c}{1 + h_p h_c} \tag{3}$$

Find an expression for the transfer function, $h_c(s)$, for the controller as a function of the ratio $\frac{y}{r}$ and the transfer function for the process, $h_p(s)$.

We will in the following subtasks specify that the set point response from the reference, r, to the output, y, should be given by

$$\frac{y}{r} = \frac{1 - \tau s}{1 + T_c s} \tag{4}$$

where T_c is a user specified time constant.

b) Assume that the process, $h_p(s)$, is modelled by a 2nd order transfer function given by

$$h_p(s) = k \frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)}, \tag{5}$$

where $T_1 > T_2 > 0$.

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

c) Assume that the process, $h_p(s)$, is modelled by a 1st order model given by

$$h_p(s) = k\frac{1-\tau s}{1+T_1 s}. \tag{6}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

d) Assume that the process, $h_p(s)$, is modelled by a 2nd order oscillating process of the form

$$h_p(s) = k\frac{1-\tau s}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \tag{7}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

e) Assume that the process is modelled by a pure time delay, i.e. with a process model

$$h_p(s) = ke^{-\tau s}. \tag{8}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

f) Assume that the process, $h_p(s)$, is modelled by a 1st order model without time delay given by

$$h_p(s) = k\frac{1}{1+T_1 s}. \tag{9}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

g) Suggest a value for the specified time constant T_c for the set point response.

Task 2 (4%):
Model reduction and the half rule

Given a 5th order process $y = h_p(s)u$ where the process transfer function, $h_p(s)$, is given by

$$h_p(s) = k\frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)(1+T_5 s)} \tag{10}$$

where $T_1 \geq T_2 \geq T_3 \geq T_4 \geq T_5 > 0$.

- Use the half rule for model reduction and find a 1st order model approximation of the form

$$h_p(s) = k\frac{1-\tau s}{1+T_1 s} \tag{11}$$

- Use the half rule for model reduction and find a 2nd order model approximation of the form

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)} \tag{12}$$

Task 3 (8%): System theory

Given a system described by a state space model

$$\dot{x} = Ax + Bu, \tag{13}$$

$$y = Dx \tag{14}$$

where $u \in \mathbb{R}^r$ is the control vector, $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^m$ is the measurements vector and $x(t_0) \in \mathbb{R}^n$ is the initial value of the state vector, which usually is assumed to be known.

a) Write down an expression for the solution $x(t)$ of the state equation (13), for $t > t_0$.

b) From the result in step 3a) above find an exact discrete time model of the form

$$x_{k+1} = \Phi x_k + \Delta u_k \tag{15}$$

Assume that the control, $u(t)$, is constant over the sampling interval. You should specify expressions for Φ and Δ in Equation (15).

c) Assume now that the state equation (13) is discretized with an explicit Euler approximation. Find a discrete time model in this case.

d) The transition matrix $\Phi = e^{A\Delta t}$ can be computed from an eigenvalue decomposition of A. Write down a formula for Φ in this case.

Task 4 (4%): PI control

Consider a PI controller

$$y = h_c(s)e, \tag{16}$$

where e is the controller input and the transfer function for the PI controller is given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}. \tag{17}$$

a) Find a continuous state space model for the PI controller of the form

$$\begin{aligned} \dot{z} &= Az + Be, \quad (18) \\ u &= Dz + Eu. \quad (19) \end{aligned}$$

Specify the parameters A, B, D and E.

b) Find a discrete time state space model for the PI controller. Use explicit Euler for the discretization and that the sampling time is Δt.