

**Partial test**  
**SCE1106 Control Theory**  
**Monday 15. October 2006**  
**kl. 13.15-15.15, Rom A190**

The test consists of 3 tasks.

The test counts  $15 = 0.5 * 30$  % of the final grade  
in SCE1106 Control with implementation.

The test consists of three pages.

Aid: paper and pen.

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## Task 1 (16%):

### PID-control, the Skogestad method

We are going to study a process described by the transfer function model

$$y = h_p(s)u. \quad (1)$$

The process are to be controlled by a controller of the form

$$u = h_c(s)(r - y). \quad (2)$$

The feedback control system is illustrated in Figure (1).

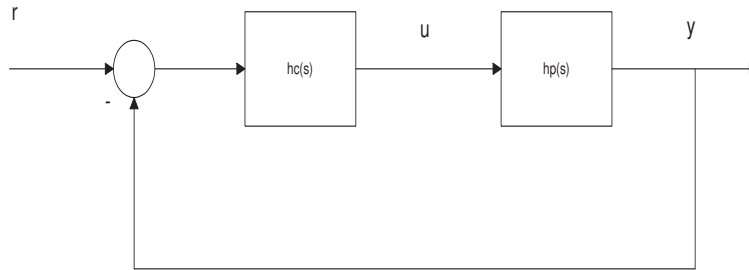


Figure 1: Standard feedback control system.

a) Consider the feedback control system in Figure (1).

- Find the transfer function from the reference,  $r$ , to the output measurement,  $y$ , i.e., find the transfer function

$$\frac{y}{r} = h_r(s) \quad (3)$$

where  $h_r(s)$  is the transfer function from  $r$  to  $y$ .

- Find an expression for the transfer function,  $h_c(s)$ , for the controller as a function of the ratio  $\frac{y}{r}$  and the transfer function for the process,  $h_p(s)$ .

We will in the following subtasks specify that the set point response from the reference,  $r$ , to the output,  $y$ , should be given by

$$\frac{y}{r} = \frac{1 - \tau s}{1 + T_c s} \quad (4)$$

where  $T_c$  is a user specified time constant.

- b) Suggest a value for the specified time constant  $T_c$  for the set point response.
- c) Assume that the process,  $h_p(s)$ , is modelled by a 2nd order transfer function given by

$$h_p(s) = k \frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)}, \quad (5)$$

where  $T_1 > T_2 > 0$ .

Find the controller  $h_c(s)$  by the Skogestad method. What type of controller is this?

- d) Assume that the process,  $h_p(s)$ , is modelled by a 1st order model given by

$$h_p(s) = k \frac{1 - \tau s}{1 + T_1 s}. \quad (6)$$

Find the controller  $h_c(s)$  by the Skogestad method. What type of controller is this?

- e) Assume that the process,  $h_p(s)$ , is modelled by a 2nd order oscillating process of the form

$$h_p(s) = k \frac{1 - \tau s}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \quad (7)$$

Find the controller  $h_c(s)$  by the Skogestad method. What type of controller is this?

- f) Assume that the process is modelled by a pure time delay, i.e. with a process model

$$h_p(s) = k e^{-\tau s}. \quad (8)$$

Find the controller  $h_c(s)$  by the Skogestad method. What type of controller is this?

- g) Assume that the process is modelled by an integrator with time delay, i.e. with a process model

$$h_p(s) = k \frac{e^{-\tau s}}{s}. \quad (9)$$

Find the controller  $h_c(s)$  by the Skogestad method. What type of controller is this?

## Task 2 (8%):

### Model reduction and the half rule

- a) Given a 5th order process  $y = h_p(s)u$  where the process transfer function,  $h_p(s)$ , is given by

$$h_p(s) = k \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)(1 + T_3 s)(1 + T_4 s)} \quad (10)$$

where  $T_1 \geq T_2 \geq T_3 \geq T_4 > 0$ .

- Use the half rule for model reduction and find a 1st order model approximation of the form

$$h_p(s) = k \frac{1 - \tau s}{1 + T_1 s} \quad (11)$$

- Use the half rule for model reduction and find a 2nd order model approximation of the form

$$h_p(s) = k \frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)} \quad (12)$$

- b) Given the process

$$h_p(s) = k \frac{e^{-\tau s}}{(1 + T_0 s)^4} \quad (13)$$

- Use the half rule for model reduction and find a 1st order model approximation of the form

$$h_p(s) = k \frac{1 - \tau s}{1 + T_1 s} \quad (14)$$

- Use the half rule for model reduction and find a 2nd order model approximation of the form

$$h_p(s) = k \frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)} \quad (15)$$

### Task 3 (6%): PID control

Consider a PID controller

$$y = h_c(s)e, \quad (16)$$

where  $e$  is the controller input and the transfer function for the PID controller is given by  $h_c(s)$ .

- a) Write the PID transfer function,  $h_c(s)$ , on cascade form.
- b) Write the PID transfer function,  $h_c(s)$ , on series form.
- c) Find the relationship between the PID controller parameters in the cascade form and the series form.